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**A practical Heckscher-Ohlin model**

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*This paper offers a formalisation of the insights of Heckscher and Ohlin that is more consistent with the evidence than standard models, and simple enough to be used for teaching and policy analysis, as well as for research. It describes both the effects of a country's factor endowments on the commodity composition of its trade and the effects of trade-related changes in goods prices on factor prices. The model applies to any numbers of goods or factors. Trade costs, including policy barriers, play a central role in the model, especially by reducing elasticities of demand.*

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## 1. Introduction

The compelling intuition of Heckscher and Ohlin – that what countries trade depends on their factor endowments and that the earnings of factors are affected by trade – combined with the elegance of Samuelson’s formalisation of this intuition has given Heckscher-Ohlin theory a prominent place in international economics. Empirically, however, the theory has had a hard time: from the 1950s to the 1980s a series of studies found little support for even its most basic proposition, which is that countries export goods that use their abundant factors intensively and import goods that use their scarce factors intensively (Baldwin, 2008, provides an excellent survey). The low point was Bowen *et al.* (1987), which decisively rejected Vanek’s (1968) factor content reformulation of Samuelson in higher dimensions.

In the 1990s, things improved for Heckscher-Ohlin (henceforth H-O). A lively debate on the causes of rising wage inequality in developed countries drew a lot of attention to H-O theory (*e.g.* Wood, 1994; Leamer, 1998). Further empirical work also yielded a clearer and less discouraging understanding of its failure of the Bowen *et al.* test. In particular, Trefler (1995) and Davis and Weinstein (2001) showed that the H-O factor content proposition is broadly correct if the assumptions of the Samuelson (HOS) and Vanek (HOV) models are modified. The key modifications are to allow for economy-wide differences in technical efficiency among countries, variation of factor prices among countries related to their endowments, and home bias in consumption.

This empirical success, however, has left economics without a practical H-O model (Deardorff, 2006; Baldwin, 2008). These modifications of the assumptions seriously undermine the HOS and HOV models. Variation of factor prices can be allowed for by having multiple cones of diversification, but at a cost in terms of complexity, both theoretically (Helpman, 1984) and empirically (Schott, 2003). Nor has HOS ever been particularly useful in higher dimensions: with more goods than factors, it implies that the composition of trade is either indeterminate or unrealistically specialised, and with more than two factors, simple general relationships between endowments and the composition of trade disappear. HOV solves these higher-dimensional problems, but at the cost of abandoning the most practically relevant contribution of H-O, which is to explain what determines the pattern of trade in actual goods.

The present paper aims to fill this gap with an alternative formalisation of Heckscher and Ohlin that is more consistent with empirical evidence, including the findings of Trefler and of Davis and Weinstein, and that works more consistently and usefully in higher dimensions. Its key innovation is to bring trade costs into the heart of the H-O model, a suggestion that recurs from Samuelson (1953: 6) to Deardorff (2006). This paper is thus in the spirit of many other recent studies of the effects of trade costs – on the directions of trade (Anderson and van Wincoop, 2004), economic geography (Henderson *et al.*, 2001), the exports of firms (Melitz, 2003; Bernard *et al.*, 2007), and the fragmentation of production (Markusen and Venables, 2007).

The main reason why trade costs matter for H-O theory is that the relative trade costs of different goods are largely independent of their relative production costs. A good that is cheaper to produce in one country than another, for example, is not necessarily cheaper to transport. As a result, the relative prices that purchasers pay for goods, which are the sum of production costs and trade costs, vary – across countries and

over time – by proportionally less than the relative prices that producers receive for them (which in equilibrium equal relative costs of production). The elasticity of demand with respect to producer prices is thus lower than the elasticity with respect to purchaser prices, to a degree that depends on the size of trade costs.

Most other models assume that relative (variable) trade costs vary in strict proportion to relative production costs. However, the idea of independent trade costs reducing demand elasticities is not new. It has been studied in industrial organisation theory, most famously by Alchian and Allen (1964), whose conjecture about ‘shipping the good apples out’ was confirmed empirically by Hummels and Skiba (2004).<sup>2</sup> What is new is to generalise this idea to all aspects of the composition of trade – across goods, not only across different qualities of goods – and to grasp its implication for any trade theory based on comparative advantage, which is that independent trade costs make producer-price elasticities of demand in open economies much lower than the infinite elasticities assumed in the HOS and HOV (and most other ‘old’ trade) models.

Less-than-infinite purchaser price elasticities in trade have been acknowledged since Armington (1969): even for commodities such as oil and grain, there are differences among supplier countries in the physical attributes of goods and conditions of supply.<sup>3</sup> Purchaser price elasticities are high, typically between 5 and 10 (Anderson and van Wincoop, 2004: 715-6), which may be close enough to infinity for some purposes. However, trade costs are large (estimated at 170% of production costs on average in developed countries by Anderson and van Wincoop, 2004), and a large share of trade costs is independent of production costs: producer-price demand elasticities are thus on average probably less than half the size of purchaser-price elasticities – between 2 and 4, which is a long way from infinite (Wood, 2008).

The technical implications of finite elasticities of demand in open economies are well understood from the many models that have used the Armington assumption – almost all CGE models and some theoretical H-O models (Robinson and Thierfelder, 1996; Venables, 2003). They make an open economy behave qualitatively like a closed one, in which factor prices vary with endowments. The present model is thus able to draw on much earlier work, and especially on that of Jones (1965). It is related to Aldaz-Carroll (2003), which adds trade costs to the Jones model in a basically similar though formally different way, and to Romalis (2004), in which factor prices also vary with endowments in a H-O model because of trade costs.

Section 2 presents the model with two goods and two factors, and explains the minor modifications that it would require to standard tools for teaching 2x2 H-O. Section 3 extends the model to any numbers of goods and factors, in a general and a simplified form. Section 4 distinguishes between exports, imports and home sales (including of non-traded goods), which involves more analysis of trade costs. Section 5 concludes with a discussion of issues that arise in applying the model empirically.

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<sup>2</sup> The conjecture was that, since the cost of transport is the same for all qualities of a good, while the cost of production rises with quality, the relative price of the better-quality varieties will be lower at the point of sale than at the point of production. Hummels and Skiba tested this conjecture, using data on trade in varying qualities of the same goods.

<sup>3</sup> Another reason is limited information, as emphasised by Rauch and Trindade (2003): purchasers are less sure that goods from foreign suppliers will be what they want.

## 2. Two goods and two factors

The low-dimensional version of the model in this section uses the framework of Jones (1965). Its algebra is similar to that of the ‘closed’ version of the Jones model, which includes a goods-demand equation. Its economic interpretation, however, is different, because it refers to an open or trading economy, in which price inelasticity of demand for goods is caused by, and varies with, trade costs.

### 2.1 Producer-price demand elasticity

Consider a single country producing its own varieties of two goods,  $j$  and 1, for sale in a single world market of which it has only small shares for both goods. Its relative sales and outputs of the two goods vary with the relative prices it charges to purchasers according to a demand function

$$\hat{q}_j - \hat{q}_1 = -\beta_{j1}(\hat{p}_j - \hat{p}_1) \quad (2.1)$$

where hats denote proportional changes,  $q_j$  is sales of good  $j$ ,  $p_j$  is the purchaser price of good  $j$ , and  $\beta_{j1}$  is an average across goods  $j$  and 1 of the elasticities of substitution of purchasers among national varieties.<sup>4</sup>

Because the relative costs of trading goods are partly independent of the relative costs of producing them, rather than proportional to relative production costs, as explained more fully in Wood (2008), relative purchaser prices vary less than relative producer prices (those at the factory gate or farm gate). More precisely,

$$\frac{\hat{p}_j - \hat{p}_1}{\hat{c}_j - \hat{c}_1} = \frac{1}{1 + \tilde{\tau}_{j1}} \quad (2.2)$$

where  $c_j$  is the producer price of good  $j$  and  $\tilde{\tau}_{j1}$  is an average of

$$\tilde{\tau}_j = \frac{\tilde{t}_j}{c_j + \tilde{t}_j} \quad (2.3)$$

and the corresponding expression for good 1, in which  $\tilde{t}_j$  is that part of the (variable) cost of trading good  $j$  that is independent of its producer price, and  $\tilde{t}_j$  is the part that is proportional to its producer price. It is assumed that all variable trade costs can be fully decomposed into these two parts and that there are no fixed trade costs.

Equation (2.2) shows how much independent trade costs damp the effects of changes in producer prices on changes in purchaser prices. To understand this equation, note that  $1/(1 + \tilde{\tau}_{j1})$  is the share of  $c + \tilde{t}$  in  $p$  (since  $p \equiv c + \tilde{t} + \tilde{\tau}$ ), averaged across goods  $j$  and 1. With no change in relative  $\tilde{t}$ ’s, this share governs the size of the effect of a change in relative  $c$ ’s, and thus relative  $(c + \tilde{t})$ ’s, on relative  $p$ ’s, which is smaller, the

<sup>4</sup> As explained in Wood (2008), this average is a weighted harmonic mean, in which the weights are inversely proportional to the relative quantities of the two goods purchased.

larger are independent trade costs. For example, if  $\tilde{\tau}_{j1}$  were unity, the share would be half, so a 10% change in relative  $c$ 's would cause only a 5% change in relative  $p$ 's.

Substituting (2.2) into (2.1) yields

$$\hat{q}_j - \hat{q}_1 = -\frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} (\hat{c}_j - \hat{c}_1) \quad (2.4)$$

which shows how the country's relative sales of goods  $j$  and 1 vary with its relative producer prices. The ratio on its right-hand side, in other words, is the *producer-price demand elasticity*, which plays a crucial role in the present model. The size of this elasticity depends partly on that of the purchaser-price elasticity,  $\beta_{j1}$ , and hence varies with the degree of substitutability among national varieties of the goods concerned, as in many models following Armington (1969). What is new about the present model is that the size of this demand elasticity also depends on, and varies with, trade costs: for goods with higher trade costs and countries with higher trade barriers, producer-price demand elasticities are lower than for goods and countries with lower trade costs.

## 2.2 Effects of changes in endowments

Assuming that internal competition causes producer prices to be equal to production costs, expansion of equation (2.4) shows how changes in relative factor prices affect the country's relative sales and outputs of the two goods

$$\hat{q}_j - \hat{q}_1 = -\frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} (\theta_{ij} - \theta_{i1}) (\hat{w}_i - \hat{w}_1) \quad (2.5)$$

where the two factors are  $i$  and 1,  $w_i$  is the price of  $i$ , and  $\theta_{ij}$  is the share of factor  $i$  in  $c_j$ , the production cost or producer price of good  $j$ . The direction of the effect of a change in factor prices depends on the sign of  $(\theta_{ij} - \theta_{i1})$ : if good  $j$  is the more factor  $i$ -intensive one, a rise in the relative price of factor  $i$  reduces relative sales of good  $j$ , to an extent governed by the sizes of  $(\theta_{ij} - \theta_{i1})$  and the producer-price demand elasticity.

Relative factor prices depend inversely on relative factor endowments, in a way to be explained shortly but which can be summarised by

$$\hat{w}_i - \hat{w}_1 = -\varphi_{i1} (\hat{v}_i - \hat{v}_1) \quad (2.6)$$

where  $\varphi_{i1}$  is signed like a substitution elasticity. Substituting (2.6) into (2.5) yields

$$\hat{q}_j - \hat{q}_1 = \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} (\theta_{ij} - \theta_{i1}) \varphi_{i1} (\hat{v}_i - \hat{v}_1) \quad (2.7)$$

which shows how a country's relative sales in the world market (and hence its outputs of) goods  $j$  and 1 vary with its relative factor endowments, in the spirit of Heckscher and Ohlin. This equation, which is similar in form to equation (11') in Jones (1965), can be interpreted causally by going from right to left: relative endowments influence

relative factor prices via  $\varphi_{i1}$ , which influence relative producer prices via  $(\theta_{ij} - \theta_{i1})$ , which influence relative outputs via  $\beta_{j1}/(1 + \tilde{\tau}_{j1})$ . The effect of relative endowments on relative outputs is simply the product of these three elasticities: its size depends on their sizes, while its direction depends on the sign of  $(\theta_{ij} - \theta_{i1})$ : a larger endowment of factor  $i$  increases the relative output of the  $i$ -intensive good.

To explain the effect of relative endowments on relative factor prices,  $\varphi_{i1}$ , requires a factor market clearing condition

$$\hat{v}_i - \hat{v}_1 = -\sigma(\hat{w}_i - \hat{w}_1) + (\lambda_{ij} - \lambda_{1j})(\hat{q}_j - \hat{q}_1) \quad (2.8)$$

where  $\lambda_{ij}$  is the share of the endowment of factor  $i$  used by good  $j$ , and

$$\sigma = \sum_{j,1} [\theta_{ij}\lambda_{1j} + (1 - \theta_{ij})\lambda_{ij}] \sigma_j \quad (2.9)$$

is a weighted average of the elasticities of substitution between factors  $i$  and 1 for the two goods,  $\sigma_j$  and  $\sigma_1$ . Equation (2.8) specifies that a rise (say) in the relative supply of factor  $i$  must be matched by a rise in relative demand, which can be achieved by an increase in the  $i$ -intensity of the techniques used for both goods (the first term), or by a shift in the sectoral composition of output towards the  $i$ -intensive good (the second term), or by some mixture of these two mechanisms. The neatly additive combination of the two mechanisms in (2.8) is accurate only for small changes, but these two basic mechanisms are the relevant ones for changes of any size. Substituting from equation (2.5) for  $(\hat{q}_j - \hat{q}_1)$  in equation (2.8) and rearranging yields

$$\hat{w}_i - \hat{w}_1 = -\frac{1}{\sigma + (\lambda_{ij} - \lambda_{1j})\frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}}(\theta_{ij} - \theta_{i1})}(\hat{v}_i - \hat{v}_1) = -\varphi_{i1}(\hat{v}_i - \hat{v}_1) \quad (2.10)$$

which shows how relative factor prices vary with relative factor endowments (and so provides a precise definition of  $\varphi_{i1}$ ).<sup>5</sup>

Equation (2.10) involves both factor market-clearing mechanisms. The second term in the denominator of the big ratio shows how a change in factor prices, by altering goods prices, shifts the sectoral composition of output in a direction that lessens the required change in factor prices. The response of relative factor prices to a change in relative endowments thus depends inversely on both the elasticity of substitution in production,  $\sigma$ , and the producer-price elasticity of demand,  $\beta_{j1}/(1 + \tilde{\tau}_{j1})$ . If national varieties were perfect substitutes, so  $\beta_{j1}$  was infinite, equation (2.10) would collapse, because factor prices would be unaffected by endowments (as in HOS). Imperfect substitutability, though, makes factor prices vary with endowments, and this variation of factor prices is amplified by independent trade costs.

Trade costs affect the relationship between outputs and endowments in equation (2.7) in two opposing ways: they reduce the response of relative sales to relative costs (and

<sup>5</sup>  $\varphi_{i1}$  is the inverse of Jones's (1965: 564) 'economy-wide elasticity of substitution between factors'.

hence to relative factor prices), but they increase the response of relative factor prices to relative endowments. To assess the net effect, the definition of  $\varphi_{i1}$  from (2.10) can be substituted into (2.7) and rearranged to yield

$$\hat{q}_j - \hat{q}_1 = \frac{1}{\frac{\sigma(1 + \tilde{\tau}_{j1})}{(\theta_{ij} - \theta_{i1})\beta_{j1}} + (\lambda_{ij} - \lambda_{1j})} (\hat{v}_i - \hat{v}_1) \quad (2.11)$$

This equation shows that in a 2x2 model, since  $(\theta_{ij} - \theta_{i1})$  and  $(\lambda_{ij} - \lambda_{1j})$  must have the same sign, trade costs make the effect of endowments on outputs smaller, whatever its direction (though this simple result does not generalise to higher dimensions). If  $\beta_{j1}$  were infinite, the big ratio in equation (2.11) would reduce to  $1/(\lambda_{ij} - \lambda_{1j})$ , which is the Rybczynski relationship in Jones (1965): changes in relative endowments would have magnified effects on relative outputs. With finite  $\beta_{j1}$ , the effects are smaller, to a degree dependent on the size of trade costs, and need not be magnified. An increased (say) endowment of factor  $i$ , by raising the output of the  $i$ -intensive good, lowers the price of that good and thus also the price of factor  $i$ , which induces  $i$ -using changes in technique that absorb some of the increased endowment.

### 2.3 Changes in foreign prices and trade costs

To analyse the effects of changes in foreign goods prices on this country, continuing to assume that it has only small market shares for both goods, the demand function in (2.1) can be expanded to

$$\hat{q}_j - \hat{q}_1 = -\beta_{j1}(\hat{p}_j - \hat{p}_1) + (\beta_{j1} - \gamma_{j1})(\hat{p}_j^* - \hat{p}_1^*) \quad (2.12)$$

where  $p_j^*$  is the average purchaser price charged by foreign suppliers of good  $j$ , and  $\gamma_{j1}$  is the elasticity of substitution in consumption between goods  $j$  and 1. The effect of a change in foreign relative prices on the relative sales of the country concerned is in the opposite direction to that of a change in its own relative prices. The effect is smaller, too, because a rise in  $p_j^*$  (for example) not only increases this country's share of the market for good  $j$  (to the same degree as a fall in  $p_j$  would) but also offsettingly shrinks the market for good  $j$  relative to good 1, to a degree determined by  $\gamma_{j1}$  (which is generally smaller than  $\beta_{j1}$ ).

By substituting equation (2.12) into equation (2.8), an expression can be derived that shows how this country's relative factor prices are affected by changes in the prices of foreign suppliers, holding its own endowments constant

$$\hat{w}_i - \hat{w}_1 = \varphi_{i1}(\lambda_{ij} - \lambda_{1j})(\beta_{j1} - \gamma_{j1})(\hat{p}_j^* - \hat{p}_1^*) \quad (2.13)$$

The change in the relative sales and outputs of the two goods caused by the change in foreign prices is translated by  $(\lambda_{ij} - \lambda_{1j})$  into a change in the relative demand for the two factors, which  $\varphi_{i1}$  translates into a change in their relative prices. As in other H-O models, a rise (say) in the relative foreign price of a good raises the relative price of the factor in which that good is intensive (which depends on the sign of  $\lambda_{ij} - \lambda_{1j}$ ).

Equation (2.13) is similar in form to (2.7), illustrating the reciprocity of H-O models. In both equations, the sign and size of the effect depend on the difference in factor intensity between the goods, measured in one equation by  $(\theta_{ij} - \theta_{i1})$  and in the other by  $(\lambda_{ij} - \lambda_{1j})$ . In both equations, too, the size of the effect is greater, the higher are the purchaser-price elasticity,  $\beta_{j1}$ , and the factor-price elasticity,  $\varphi_{i1}$ .

There is an asymmetry between equations (2.7) and (2.13) in the effect of trade costs: in both equations, trade costs increase  $\varphi_{i1}$  (from equation 2.10), but in (2.7) trade costs also lower the goods-price demand elasticity. The net effect of trade costs is thus to reduce the response of outputs to changes in endowments (as shown in 2.11), but to increase the response of factor prices to changes in foreign prices (though this simple asymmetry is another result that does not generalise).

To analyse more deeply the response of factor prices to foreign prices, the expression for  $\varphi_{i1}$  from (2.10) can be substituted into (2.13) and rearranged to yield

$$\hat{w}_i - \hat{w}_1 = \frac{1}{\frac{\sigma}{(\lambda_{ij} - \lambda_{1j})\beta_{j1}} + \frac{(\theta_{ij} - \theta_{i1})}{(1 + \tilde{\tau}_{j1})}} (1 - \gamma_{j1}/\beta_{j1})(\hat{p}_j^* - \hat{p}_1^*) \quad (2.14)$$

If  $\beta_{j1}$  were infinite and  $\tilde{\tau}_{j1}$  were zero, the right-hand side of (2.14) would reduce to  $(\hat{p}_j^* - \hat{p}_1^*)/(\theta_{ij} - \theta_{i1})$ , which is the Stolper-Samuelson relationship in Jones (1965): the effect of a change in foreign prices on factor prices would be magnified. A finite  $\beta_{j1}$  tends to reduce this effect, because of the feedback of a change in factor prices via changes in technique and thus in factor availabilities, outputs and goods prices (in the first term of the denominator of the big ratio in 2.14). Independent trade costs, by contrast, tend to increase this effect, fundamentally because the shock is to purchaser prices, changes in which, with independent trade costs, require proportionally bigger changes in producer prices and hence in factor prices.

A change in foreign relative prices may be caused by a change in foreign production costs,  $c^*$ , or in trade costs on foreign goods,  $t^*$ , including policy barriers such as tariffs (not distinguishing here between independent and proportional trade costs)

$$\hat{p}_j^* - \hat{p}_1^* = \hat{c}_j^* - \hat{c}_1^* + \hat{T}_j^* - \hat{T}_1^* \quad (2.15)$$

where  $T_j^* = 1 + t_j^*/c_j^*$ , and similarly for good 1. For example, to show the effects of a change in relative foreign trade costs, assuming no change in foreign production costs, the final term  $(\hat{p}_j^* - \hat{p}_1^*)$  in equation (2.13) or (2.14) is replaced by  $(\hat{T}_j^* - \hat{T}_1^*)$ . As usual in H-O models, a reduction (say) in the trade costs of foreign suppliers of a good would lower the price of the factor used intensively in the production of that good.

To analyse the effects of changes in this country's own trade costs, equation (2.2) can be expanded to



$$\hat{p}_j - \hat{p}_1 = \frac{1}{1 + \tilde{\tau}_{j1}} \left( \hat{c}_j - \hat{c}_1 + \hat{T}_j - \hat{T}_1 \right) + \frac{\tilde{\tau}_{j1}}{1 + \tilde{\tau}_{j1}} \left( \hat{t}_j - \hat{t}_1 \right) \quad (2.16)$$

where

$$\ddot{T}_j = 1 + \ddot{\tau}_j = 1 + \ddot{t}_j / c_j$$

and similarly for good 1. Changes in relative purchaser prices are a weighted sum of changes in relative producer prices plus proportional trade costs ( $\ddot{t}$ ) and changes in relative independent trade costs ( $\tilde{t}$ ), where the weights are their shares in purchaser prices. Changes in one-plus-proportional trade cost ratios ( $\ddot{T}$ s) have the same effect on purchaser prices as changes in producer prices, and thus the same weight, which is the average share of  $(c + \ddot{t})$  in  $p$ . The effect of changes in relative independent trade costs has a different weight, which is the average share of  $\tilde{t}$  in  $p$ .

Equation (2.16) can be used to derive the effect on relative factor prices of changes in own-trade costs, holding relative factor endowments constant,

$$\hat{w}_i - \hat{w}_1 = -\varphi_{i1} (\lambda_{ij} - \lambda_{1j}) \beta_{j1} \left[ \frac{1}{1 + \tilde{\tau}_{j1}} \left( \hat{T}_j - \hat{T}_1 \right) + \frac{\tilde{\tau}_{j1}}{1 + \tilde{\tau}_{j1}} \left( \hat{t}_j - \hat{t}_1 \right) \right] \quad (2.17)$$

The term in square brackets shows how changes in the country's relative trade costs affect its relative purchaser prices, and the terms before the square bracket show how this price change affects relative sales of the two goods ( $\beta_{j1}$ ), relative factor demands ( $\lambda_{ij} - \lambda_{1j}$ ) and relative factor prices ( $\varphi_{i1}$ ). The sign is negative: a fall (say) in own-trade costs on the  $i$ -intensive good (trade taxes as well as things like transport costs) raises the relative price of factor  $i$ , unlike a fall in the trade costs of foreign suppliers of the  $i$ -intensive good, which as shown above would hurt factor  $i$ .

Allowing for the endogeneity of  $\varphi_{i1}$ , equation (2.17) can be rewritten as<sup>6</sup>

$$\hat{w}_i - \hat{w}_1 = - \frac{1}{\frac{\sigma(1 + \tilde{\tau}_{j1})}{(\lambda_{ij} - \lambda_{1j})\beta_{j1}} + (\theta_{ij} - \theta_{i1})} \left[ \left( \hat{T}_j - \hat{T}_1 \right) + \tilde{\tau}_{j1} \left( \hat{t}_j - \hat{t}_1 \right) \right] \quad (2.18)$$

If  $\beta_{j1}$  were infinite and  $\tilde{\tau}_{j1}$  were zero, the right-hand side of (2.18) would reduce to  $-\left( \hat{T}_j^* - \hat{T}_1^* \right) / (\theta_{ij} - \theta_{i1})$ , implying magnification. With finite  $\beta_{j1}$ , the effects of changes in trade costs on factor prices are smaller, because there is some shifting of trade costs onto purchasers. Higher independent trade costs,  $\tilde{\tau}_{j1}$ , lessen the effect of changes in

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<sup>6</sup> Equation (2.18) differs from equation (2.14) in the position of the  $(1 + \tilde{\tau}_{j1})$  term because the final square-bracketed term in (2.18) refers to changes in relative producer prices, whereas the final term in (2.14) refers to changes in relative purchaser prices.

proportional trade costs,  $\hat{T}_j^* - \hat{T}_1^*$  (though again this result does not generalise), but could either raise or lower the effect of changes in independent trade costs,  $\hat{t}_j - \hat{t}_1$ .

#### 2.4 Teaching the practical 2x2 model

The main use of the 2x2 version of H-O is for teaching, so it is worth pausing, before moving to higher dimensions, to consider how the model described above might alter the way in which 2x2 H-O is taught. Since the basic economic insights remain the same, as does much of the model, the same teaching tools could be used, albeit with some significant modifications of emphasis and interpretation.

The model above is of one small country trading in a world market, which for some purposes is sufficient, but the teaching of trade clearly has to involve more than one country. The hatted changes in variables thus need to be interpreted as referring to differences across countries as well as to movements over time in one country. The algebra strictly applies only to small changes and hence also only to small differences, but it will be argued below that the model can be extended with acceptable accuracy to large changes and differences by working in logs.

Extending the model to many countries requires some additional assumption about technology, of which the obvious one is that all countries have the same technology up to a scale factor reflecting economy-wide efficiency as a result of variation in, say, the quality of institutions. This assumption allows the mix of factors in each sector to vary with endowments. It is weaker than the usual HOS one of identical technology, and conforms with the results of Trefler (1995) and Davis and Weinstein (2001). This assumption also underlines that H-O is a theory of comparative advantage.

The usual H-O theorems all continue to apply, though in less strong and simple forms than in HOS. Rybczynski and Stolper-Samuelson were discussed above, and so was reciprocity. Changes in relative endowments still act in the usual direction on relative outputs, as do changes in relative foreign prices on relative factor prices. What is different is the magnitudes of these effects. However, the strong and simple theorems of HOS remain useful for explaining the essence of H-O (Leamer, 1995).

Factor price equalisation exists only in special cases: relative factor prices vary among countries with endowments, and absolute factor prices vary also because of the scale differences in technology. What remains is relative factor price convergence, which is what was predicted by Ohlin (1967: 24-8). If reduced trade costs cause a country's relative goods prices to move towards the relative goods prices of other countries, its relative factor prices will also tend to move towards their relative factor prices.

The H-O theorem – that a country's exports use its abundant factor intensively, and vice versa for its imports – can be illustrated in the model by interpreting the changes in equations (2.7) or (2.11) as referring to deviations of a country's endowment ratio from the world average endowment ratio. What endowments alter in the model above is the structure of production, from which the effects on exports and imports follow with the usual additional assumption that consumption preferences are similar across countries. In section 4, exports and imports are modelled separately.

Many of the same diagrams could also still be used. An important one, though not specifically H-O, illustrates the gains from trade with a production possibility frontier, indifference curves and a world price ratio line. Two modifications are required. One is to have two world price ratio lines: the relative foreign cost of production, and the relative purchaser price derived by raising this relative cost to the power  $1/(1 + \tilde{\tau}_{j1})$ . The other modification is to increase the curvature of the indifference curves: the production possibility frontier refers to the varieties of the two goods produced by the country concerned, for which the varieties obtained for consumption through imports are less-than-perfect substitutes.<sup>7</sup>

The invaluable Lerner diagram can initially be presented in the usual way and used to illustrate the effects of a shift in country endowments on the inter-sectoral allocation of factors. It can then be explained that shifts in the sectoral composition of output usually require opposite shifts in relative goods prices and thus in the distances of the unit value isoquants from the origin (since the isoquants are valued at producer prices, independent trade costs amplify these shifts). Relative factor prices thus move in the opposite direction to the endowment ratio, and the factor intensity ratios of both goods move in the same direction as the endowment ratio, which makes the inter-sectoral reallocation of the factors smaller than if goods prices were unchanged.<sup>8</sup>

Much of trade can be taught without simultaneous consideration of more than one country, and analysing a realistic world of many countries requires simulation. Two-country models are also useful, though, including in teaching H-O. A variant of the integrated equilibrium diagram of Dixit and Norman (1980), without factor price equalisation but with only two goods, could be a two-country Lerner diagram. The factor intensity ratios of the goods differ between the two countries because of (and in the same direction) as the differences in their endowment ratios. The endowments of each country, shown by a single point for both countries, are allocated between the goods as in any Lerner diagram, and the country allocations are summed in another point to show how world factor supplies are divided between the two goods.

Two-country North-South models are also useful (*e.g.* Wood, 1994). In this context, the easiest way to apply the present algebra is to assume that each region contains many identical countries and to focus on representative ones in each region. It is then simple to analyse the effects of, say, a change in the South's own relative trade costs on relative wages both in the South (equation 2.17) and in the North (equation 2.13), since the change in the South's trade costs also alters foreign prices in the North. The outcome may be modified by technical change, either autonomous or caused by more openness to trade, whose effects on factor prices in a model of this type are analysed by Jones (1965). The relative effects of sector-biased and factor-biased change differ from those in the usual HOS model with infinitely elastic demand for goods.

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<sup>7</sup> With trade, consumers choose at the margin between the home variety of the export good and foreign varieties of the imported good: the elasticity of substitution is roughly  $\gamma_{j1}(1 - 1/\beta_{j1})$ , which gets closer to  $\gamma_{j1}$ , the more substitutable are home and foreign varieties. There is more curvature in an open than in a closed economy, which reduces the gain from trade, but not by enough to make it worth drawing.

<sup>8</sup> Interpreted literally, the present model also implies that the endowment ratio can never lie outside the range of the factor intensity ratios. Since each country produces its own variety, production in each of its sectors can never quite fall quite to zero. It is more instructive, however, to continue to interpret the diagram as implying complete specialisation when the endowment ratio is outside this range.

### 3. Any numbers of goods or factors

This section extends the 2x2 model to higher dimensions. Goods are indexed by the subscript  $j$  ( $= 1, \dots, n$ ) and factors by  $i$  or  $k$  ( $= 1, \dots, m$ ). Following Smith and Wood (2009), a standard higher-dimensional Jones-type H-O model contains  $n - 1$  equations linking changes in relative goods prices to changes in relative factor prices

$$\hat{p}_j - \hat{p}_1 = \sum_{k=2}^m (\theta_{kj} - \theta_{k1})(\hat{w}_k - \hat{w}_1) \quad (3.1)$$

of which all the terms were defined in the previous section, and  $m - 1$  factor market-clearing equations, also in changes,

$$\hat{v}_i - \hat{v}_1 = -\sum_{k=2}^m \left[ \sum_{j=1}^n (\lambda_{ij} \sigma_{ijk} - \lambda_{1j} \sigma_{1,jk}) \right] (\hat{w}_k - \hat{w}_1) + \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j})(\hat{q}_j - \hat{q}_1) \quad (3.2)$$

which are of the same general form as the factor market-clearing equation in the 2x2 model, but require a bit more explanation. The last term differs only in summing the effects on relative factor demands of changes in relative outputs over  $n - 1$  pairs of goods, rather than one pair. The first term on the right-hand side, however, involves a redefinition: the  $\sigma$  terms are no longer elasticities of substitution: instead

$$\sigma_{ijk} = -\frac{\partial a_{ij}}{\partial w_k} \frac{w_k}{a_{ij}} \quad (3.3)$$

is the elasticity with respect to the price of factor  $k$  of the factor  $i$  input coefficient for good  $j$ . The partial derivative in (3.3) can be of either sign, but for consistency with the previous section (and differently from Smith and Wood, 2009),  $\sigma_{ijk}$  is defined like a substitution elasticity: if  $\partial a_{ij} / \partial w_k$  is negative,  $\sigma_{ijk}$  is positive. These elasticities are weighted by the shares of the endowment of  $i$  used in good  $j$ , and summed over all pairs of goods and factors. The first right-hand side term of (3.2) thus shows how changes in all relative factor prices affect the relative demands for factors  $i$  and 1.

Equations (3.1) and (3.2) would normally be interpreted as referring to an economy facing exogenous goods prices,  $p_j$ . As in the previous section, however, independent trade costs can be introduced into (3.1), which becomes

$$\hat{p}_j - \hat{p}_1 = \frac{1}{1 + \tilde{\tau}_{j1}} (\hat{c}_j - \hat{c}_1) = \frac{1}{1 + \tilde{\tau}_{j1}} \sum_{k=2}^m (\theta_{kj} - \theta_{k1})(\hat{w}_k - \hat{w}_1) \quad (3.4)$$

and another equation can be added

$$\hat{q}_j - \hat{q}_1 = -\sum_{j=2}^n \beta_{j1} (\hat{p}_j - \hat{p}_1) \quad (3.5)$$

which makes the country's relative sales and thus outputs of goods  $j$  and 1 depend on the relative purchaser prices of its varieties of these and all other goods. Combining equations (3.4) and (3.5) yields  $n - 1$  equations of the form

$$\hat{q}_j - \hat{q}_1 = - \sum_{j=2}^n \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} \left[ \sum_{k=2}^m (\theta_{kj} - \theta_{k1})(\hat{w}_k - \hat{w}_1) \right] \quad (3.6)$$

which in conjunction with the  $m - 1$  factor market-clearing equations (3.2) describe how changes in all relative factor prices and relative outputs depend on changes in the exogenous relative factor endowments. Changes in the relative purchaser prices of all goods are determined by equations (3.4). Expansion of (3.5) along the lines of (2.12) and (2.16) would allow analysis of the effects on factor prices and outputs of changes in foreign goods prices and trade costs. As in its 2x2 version, the model depends on the  $\beta_{j1}$ s being finite: if different national varieties of goods were perfect substitutes, it would collapse back to the standard system of equations (3.1) and (3.2).

This model has the basic H-O feature of showing how the composition of a country's output is affected by the composition of its factor endowments, but has three notable advantages over most higher-dimensional H-O models. It applies to (and behaves in the same way with) any numbers of goods and factors: these numbers do not have to be even, and nor does the model become indeterminate if, as in reality, the number of goods exceeds the number of factors. A second advantage is that relative factor prices are always influenced by relative endowments, as they appear to be in reality but are not in the usual one-cone HOS model. A third advantage is that this model shows how outcomes are affected by trade costs.

### 3.1 Simplification of the model

In these three respects, this general higher-dimensional version of the present model can claim to be practical. What is not practical about it, however, like other higher-dimensional H-O models, is that it is complicated and that changes in endowments could in principle affect outputs and factor prices in almost any way. To get a more tractable model and clearer results, as Bliss (2007: 128) emphasises, it is necessary to make simplifying assumptions. The rest of this section will therefore formulate and analyse a simpler version of the present higher-dimensional model.

A first simplification is to omit the effects on a country's relative sales of any pair of (its varieties of) goods of the prices of (its varieties of) other goods: it seems unlikely that choices between (say) French cars and French shirts depend much on the price of French shoes in particular. Each of the  $n - 1$  equations (3.6) thus becomes

$$\hat{q}_j - \hat{q}_1 = - \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} \sum_{k=2}^m (\theta_{kj} - \theta_{k1})(\hat{w}_k - \hat{w}_1) \quad (3.7)$$

which can be substituted into equations (3.2) to yield

$$\hat{v}_i - \hat{v}_1 = -\sum_{k=2}^m \left[ \sum_{j=1}^n (\lambda_{ij} \sigma_{ijk} - \lambda_{1j} \sigma_{1jk}) \right] (\hat{w}_k - \hat{w}_1) - \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j}) \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} \sum_{k=2}^m (\theta_{kj} - \theta_{k1}) (\hat{w}_k - \hat{w}_1) \quad (3.8)$$

A much more radical simplification is then to convert each of the equations (3.8) into a relationship between the relative prices and endowments of a single pair of factors,  $i$  and 1. Algebraically, this involves suppressing the two summations over  $k$ : in the first right-hand side term, it omits the effects on the choice of technique as between factors  $i$  and 1 of the prices of all factors other than  $i$  and 1; in the second term, it omits the effects on the relative prices, and hence the relative outputs, of goods  $j$  and 1 of the relative prices of all factors other than  $i$  and 1. With these simplifications, (3.8) can be rearranged to yield  $m - 1$  elasticities,  $\varphi_{i1}$ , that show how each pair of relative factor prices depends, inversely, on the corresponding pair of factor endowments

$$\varphi_{i1} = -\frac{\hat{w}_i - \hat{w}_1}{\hat{v}_i - \hat{v}_1} = \frac{1}{\sum_{j=1}^n (\lambda_{ij} \sigma_{iji} - \lambda_{1j} \sigma_{1ji}) + \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j}) \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} (\theta_{ij} - \theta_{i1})} \quad (3.9)$$

The elasticity  $\varphi_{i1}$  was used above in the 2x2 version, and equation (3.9) is similar to (2.10), except that it involves summations over all pairs of goods (underlining that  $\varphi_{i1}$  is an economy-wide relationship).<sup>9</sup> The  $k$  subscript is dropped, since only two factors are involved in each equation, but the triple subscript on  $\sigma_{iji}$  is retained as a reminder of one of the simplifications made in deriving this expression.

As with all simplification of models, the benefit of these simplifications of (3.8) needs to be weighed against their cost. The benefit, a much clearer higher-dimensional H-O model, will emerge in the next sub-section. The cost, a reduction in the accuracy of the model, will then be explained and discussed.

### 3.2 Higher-dimensional H-O relationships

Using (3.9), equation (3.7) can be extended into

$$\hat{q}_j - \hat{q}_1 = \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} \sum_{i=2}^m (\theta_{ij} - \theta_{i1}) \varphi_{i1} (\hat{v}_i - \hat{v}_1) \quad (3.10)$$

a set of  $n - 1$  equations showing how the relative outputs of all pairs of goods depend on the relative endowments of all pairs of factors. These equations are similar to (2.7) in the 2x2 version of the model, except that there are more of them and that each of them involves many pairs of factors rather than only one pair. Their meaning can be brought out by writing the equation for (say) good 2 as a series of  $m - 1$  terms, each of which is a change in one factor endowment ratio multiplied by a coefficient

<sup>9</sup> The expression being summed in the second term in the denominator of (3.9) may not be positive for all pairs of goods, since in higher dimensions the  $\lambda$  and  $\theta$  difference terms may not have the same sign, but its sum over all pairs of goods, and hence (3.9) as a whole, seems certain to be positive.

$$\hat{q}_2 - \hat{q}_1 = \frac{\beta_{21}}{1 + \tilde{\tau}_{21}} (\theta_{22} - \theta_{21}) \varphi_{21} (\hat{v}_2 - \hat{v}_1) + \dots + \frac{\beta_{21}}{1 + \tilde{\tau}_{21}} (\theta_{m2} - \theta_{m1}) \varphi_{m1} (\hat{v}_m - \hat{v}_1) \quad (3.11)$$

The negative signs on  $\beta_{21}$  and the  $\varphi_{i1}$ 's cancel, so the sign of each coefficient depends on that of its  $(\theta_{i2} - \theta_{i1})$  term: if good 2 uses factor  $i$  more intensively than good 1, the coefficient is positive, and if less intensively, is negative. The  $(\theta_{i2} - \theta_{i1})$  term affects also the size of the coefficient: the bigger the difference between goods 1 and 2 in the intensity of their use of factor  $i$ , the larger is the coefficient (and if the two goods were of equal factor intensity, the term would vanish). The size of each coefficient depends also on the size of the relevant  $\varphi_{i1}$ , which can vary among pairs of factors, depending on the parameters of equation (3.9).

All the coefficients in (3.11) depend in the same way on  $\beta_{21}/(1 + \tilde{\tau}_{21})$ : the lower is the average degree of substitutability among varieties of goods 1 and 2, and the higher are independent trade costs on these two goods, the smaller are the effects of all pairs of factor endowments on the relative outputs of goods 1 and 2. Independent trade costs pull in the other direction, too, by making the  $\varphi_{i1}$ s larger – amplifying the effects of endowments on factor prices. This amplification depends not on  $\tilde{\tau}_{21}$ , however, but (from 3.9) on average  $\tilde{\tau}_{j1}$ s across all pairs of goods. The net effect of a higher  $\tilde{\tau}_{21}$ , holding other  $\tilde{\tau}_{j1}$ s constant, is to reduce the response of the relative outputs of goods 1 and 2 to endowments (as in the 2x2 version). Raising other  $\tilde{\tau}_{j1}$ s, by contrast, makes the relative outputs of goods 1 and 2 more responsive to endowments.

The assumptions used in deriving equation (3.9) also facilitate analysis of the effects of changes in foreign prices and own trade costs on relative factor prices, as in the 2x2 model. Considering first changes in foreign purchaser prices, which could be caused by changes in trade costs on (including policy barriers to) foreign goods, the demand equations (3.7) become

$$\hat{q}_j - \hat{q}_1 = -\frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}} \sum_{k=2}^m (\theta_{kj} - \theta_{k1}) (\hat{w}_k - \hat{w}_1) + (\beta_{j1} - \gamma_{j1}) (\hat{p}_j^* - \hat{p}_1^*) \quad (3.12)$$

which can be substituted into the factor market-clearing equation and simplified as before, holding endowments constant, to yield, after rearrangement, a set of  $m - 1$  equations which describe how each pair of factor prices is affected by changes in the relative foreign purchaser prices of all pairs of goods

$$\hat{w}_i - \hat{w}_1 = \varphi_{i1} \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j}) (\beta_{j1} - \gamma_{j1}) (\hat{p}_j^* - \hat{p}_1^*) \quad (3.13)$$

Equations (3.13) can again be explained by writing out one of them (for factors 1 and 2, say) as a series of  $n - 1$  terms, each of which is the change in one foreign price ratio multiplied by a coefficient, as in the corresponding 2x2 equation (2.13)

$$\hat{w}_2 - \hat{w}_1 = \varphi_{21}(\lambda_{22} - \lambda_{12})(\beta_{21} - \gamma_{21})(\hat{p}_2^* - \hat{p}_1^*) + \dots + \varphi_{21}(\lambda_{2n} - \lambda_{1n})(\beta_{n1} - \gamma_{n1})(\hat{p}_n^* - \hat{p}_1^*) \quad (3.14)$$

The proximate effect of a change in a relative foreign price is to alter the relative sales and outputs of this country's varieties of the two goods, to an extent determined by the size of the purchaser-price elasticity,  $(\beta_{j1} - \gamma_{j1})$ . This change in relative outputs alters the relative demand for factors 1 and 2, in a direction and to an extent governed by the difference between the shares of factor 2 used by the two goods,  $(\lambda_{2j} - \lambda_{1j})$ , whose sign determines that of the coefficient. The size of each coefficient depends also on that of  $\varphi_{21}$ , common to all the coefficients.

Trade costs act through  $\varphi_{21}$ : the higher the average level of (independent) trade costs across all goods in the country concerned, the larger is  $\varphi_{21}$  and hence the more do the relative prices of factors 1 and 2 respond to a change of given size in relative demand for these two factors. Equation (3.13) thus implies that higher trade costs increase the effect of changes in foreign prices on factor prices. This implication should be treated cautiously, though, and is modified below, because equation (3.13), like all the others so far, assumes that the country has only small market shares for all its goods

The reciprocity of H-O models can be seen from the similarity of form between the quantity equations (3.10) and the price equations (3.13). Each of these equations is a series of terms involving a coefficient which includes a price elasticity common to all the terms (in the quantity equations a goods-price elasticity and in the price equations a factor-price elasticity). Each coefficient also includes a measure of difference in factor use between goods (based on  $\theta$ s in the quantity equation and on  $\lambda$ s in the price equation) and another price elasticity which varies among the terms (in the quantity equations a factor-price elasticity and in the price equations a goods-price elasticity).

To analyse the effects of changes in relative own-trade costs, equation (2.16) can be adapted and used to yield

$$\hat{w}_i - \hat{w}_1 = -\varphi_{i1} \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j}) \beta_{j1} \left[ \frac{1}{1 + \tilde{\tau}_{j1}} (\hat{T}_j - \hat{T}_1) + \frac{\tilde{\tau}_{j1}}{1 + \tilde{\tau}_{j1}} (\hat{t}_j - \hat{t}_1) \right] \quad (3.15)$$

whose form is similar to that of the foreign-price equation (3.13), apart from its final terms, but with the opposite sign. A rise in the foreign price of a good, caused say by an increased tariff, raises the price of the factor that the good uses intensively, while a rise in own-trade costs on that good lowers the price of that factor.

### 3.3 Cost of the simplifications

Equations (3.9), (3.10), (3.13) and (3.15) are basic H-O relationships: for any numbers of goods and factors, they show how the composition of a country's output and its relative factor prices are influenced by the composition of its endowments, trade costs and foreign prices. They also make these relationships far clearer than in the general higher-dimensional version of the model in equations (3.2) and (3.6). This clarity was achieved, however, by radical simplification of the factor market-clearing equations (3.8), omitting (a) the effects on the choice of technique as between factors  $i$  and 1 of



the prices of all factors other than  $i$  and 1, and (b) the effects on the relative prices of goods  $j$  and 1 of the relative prices of factors other than  $i$  and 1 (though these effects remain in other parts of the model, such as equations 3.10). The simplified version of the model is thus less accurate than the general version.

The inaccuracy can be explained by considering, say, the first term in equation (3.11)

$$\hat{q}_2 - \hat{q}_1 = \frac{\beta_{21}}{1 + \tilde{\tau}_{21}} (\theta_{22} - \theta_{21}) \varphi_{21} (\hat{v}_2 - \hat{v}_1) + \dots \quad (3.16)$$

in the case of an increase in the endowment of factor 2. The expression for  $\varphi_{21}$  in (3.9), and more specifically the second term in its denominator, is inaccurate because a fall in the relative price of factor 2 as a result of its increased supply would also alter the relative prices of goods other than 1 and 2, so the fall in the relative price of factor 2 could be larger or smaller than (3.9) implies. Moreover, increasing the endowment of factor 2 would lower the prices of other factors for which it was a substitute (and raise the prices of factors for which it was a complement), which would affect the relative production costs of goods 1 and 2, and thus their relative purchaser prices and relative outputs. Equation (3.16) should thus be expanded to something like

$$\hat{q}_2 - \hat{q}_1 = \frac{\beta_{21}}{1 + \tilde{\tau}_{21}} \left[ (\theta_{22} - \theta_{21}) \varphi_{21} + \sum_{i=3}^m (\theta_{i2} - \theta_{i1}) \varphi_{i2} \right] (\hat{v}_2 - \hat{v}_1) + \dots \quad (3.17)$$

in which the added summation could be of either sign, since the  $\varphi_{i2}$ s could be of either sign (though most would probably be positive, since substitutability is more common than complementarity) and so could the  $(\theta_{i2} - \theta_{i1})$ s.

For these reasons, the effect of a change in the relative endowments of any given pair of factors on the relative outputs of any given pair of goods is not exactly as specified in equations (3.9) and (3.10): it could be either larger or smaller. Much the same is true of the relationships in equations (3.13) between changes in relative foreign prices and in relative factor prices, because of the inaccuracy of the  $\varphi_{i1}$  terms. Shifts in the relative demand for factors 1 and 2 caused by changes in relative foreign prices could have larger or smaller effects on the prices of factors 1 and 2 than equations (3.13) and (3.9) specify. This is because changes in the prices of these two factors affect the prices of other factors for which they are substitutes or complements, which has indirect effects on the relative demands for, and prices of, these two.

The costs of these inaccuracies, relative to the benefits of the simplified version of the higher-dimensional model, depend on the purpose for which the model is being used. The inaccuracies of the simplified equations, and the more accurate general version of the model in equations (3.2) and (3.6), should not be forgotten, and for some uses, the general version is preferable. But for most practical purposes – of analysis, teaching, estimation and application to actual country cases – the simplified version is likely to be more useful, provided that readers or students are warned of its limitations and that econometric results are interpreted with due caution.

#### 4. Exports, home sales and imports

All the analysis so far has been of one country producing and selling (its varieties of) goods in a single world market (including its domestic market) of which it has small shares for all goods. This section distinguishes among different sorts of markets, in which outcomes differ mainly because trade costs differ (assuming away differences in tastes, as in most H-O models). This extension of the model is needed for explicit analysis of the two basic elements of international trade, exports and imports.

Some refinements of notation are required. Countries of origin (or supplier countries) are indexed by a superscript  $z$  ( $= 1, \dots, Z$ ), and markets (or countries of destination) by a second superscript,  $\check{z}$  ( $= 1, \dots, \check{Z}$ ). For example, country  $z$ 's sales of good  $j$  in market  $\check{z}$  are  $q_j^{z\check{z}}$ , and its purchaser price in that market is  $p_j^{z\check{z}}$ , but its producer price, common to all its markets, is just  $c_j^z$ . The superscript  $*$  can be substituted for either  $z$  or  $\check{z}$  to refer to the rest of the world as a whole: for example, the country's exports are  $q_j^{z*}$ . Trade costs are split between internal costs, incurred by all suppliers to a market, with superscript  $D$ , and international costs, incurred only by foreign suppliers, with superscript  $F$ . For example,  $t_j^{Fz*}$  is the variable international trade cost of exports of a unit of good  $j$  from country  $z$ , averaged over all its export markets.

##### 4.1 Producer-price demand elasticities

What varies among markets is the producer-price elasticity of demand, which in the two previous sections was  $\beta_{j1}/(1 + \tilde{\tau}_{j1})$ . The general form of this elasticity is

$$\varepsilon_{j1}^{z\check{z}} = \tilde{\varepsilon}_{j1}^{z\check{z}} \delta_{j1}^{z\check{z}} \quad (4.1)$$

where  $\tilde{\varepsilon}_{j1}^{z\check{z}}$  is the purchaser-price elasticity, which so far has been  $\beta_{j1}$ , and

$$\delta_{j1}^{z\check{z}} = \frac{\hat{p}_j^{z\check{z}} - \hat{p}_1^{z\check{z}}}{\hat{c}_j^z - \hat{c}_1^z} = \frac{1}{1 + \tilde{\tau}_{j1}^{Dz\check{z}} + \tilde{\tau}_{j1}^{Fz\check{z}}} \quad (4.2)$$

is the 'price-ratio elasticity', which so far has been  $1/(1 + \tilde{\tau}_{j1})$ . Both the ingredients of the producer-price elasticity in equation (4.1) have  $z$  and  $\check{z}$  superscripts, because both of them vary with trade costs. The reason for variation of the price-ratio elasticity is obvious, namely that the ratio of (independent) trade costs to other costs,  $\tilde{\tau}_{j1}^{Dz\check{z}} + \tilde{\tau}_{j1}^{Fz\check{z}}$ , depends on characteristics of both the supplier and the market.

The reason for variation of the purchaser-price elasticity requires more explanation. The response of relative sales of a country's varieties of two goods to changes in its relative purchaser prices depends in principle not only on the degree of substitutability among different varieties of each good, measured by  $\beta_{j1}$ , but also on the degree of substitutability between the two goods, measured by  $\gamma_{j1}$ . The relative impact of these two sorts of substitutability depends on the country's shares of the market for the two goods: if its shares are small,  $\tilde{\varepsilon}_{j1}^{z\check{z}}$  is determined almost entirely by  $\beta_{j1}$ , but with larger

shares,  $\tilde{\varepsilon}_{j1}^{z\tilde{z}}$  depends also on  $\gamma_{j1}$ , since the prices charged by the country in question for its varieties then have a significant effect on the average prices of the goods.

More precisely, as in Wood (2008), following Sato (1967), with CES preferences  $\tilde{\varepsilon}_{j1}^{z\tilde{z}}$  is a weighted harmonic mean of  $\beta_j$ ,  $\beta_1$  and  $\gamma_{j1}$ , in which the weights include country  $z$ 's shares of market  $\tilde{z}$  for the two goods. A more tractable approximation for purposes of exposition is the weighted arithmetic mean

$$\tilde{\varepsilon}_{j1}^{z\tilde{z}} = s_{j1}^{z\tilde{z}}\gamma_{j1} + (1 - s_{j1}^{z\tilde{z}})\beta_{j1} = \beta_{j1} - s_{j1}^{z\tilde{z}}(\beta_{j1} - \gamma_{j1}) \quad (4.3)$$

where  $\beta_{j1}$  is an average of  $\beta_j$  and  $\beta_1$ , and  $s_{j1}^{z\tilde{z}}$  is an average of country  $z$ 's shares of market  $\tilde{z}$  for goods  $j$  and 1. If  $s_{j1}^{z\tilde{z}} \approx 0$ ,  $\tilde{\varepsilon}_{j1}^{z\tilde{z}} \approx \beta_{j1}$ , as before, while if  $s_{j1}^{z\tilde{z}} = 1$ , as in a closed economy or for two non-traded goods,  $\tilde{\varepsilon}_{j1}^{z\tilde{z}} = \gamma_{j1}$ . Assuming preferences to be identical and homothetic avoids the need for  $\tilde{z}$  superscripts on  $\beta_{j1}$  and  $\gamma_{j1}$ .

Trade costs therefore influence purchaser-price elasticities by affecting market shares. The relationship is analysed in Wood (2008), but its essence can be conveyed by

$$s_{j1}^{z\tilde{z}} = s_{j1}^{z\tilde{z}}(t_{j1}^{Fz\tilde{z}} - t_{j1}^{Fz\tilde{z}}) \quad s_{j1}^{z\tilde{z}}(\cdot) > 0 \quad (4.4)$$

This equation says that country  $z$ 's average share of market  $\tilde{z}$  for goods  $j$  and 1 rises with the international trade costs incurred by the rest of the world in supplying market  $\tilde{z}$ , relative to those that country  $z$  incurs. In its home market, country  $z$  has a trade cost advantage, since  $t_{j1}^{Fz\tilde{z}} = 0$ , which tends to raise its market share, especially if natural or policy barriers to imports are high. In export markets, country  $z$  will always be at a disadvantage relative to local suppliers, and may be at an advantage or a disadvantage compared to the average of other foreign suppliers.

Note that the trade cost terms in (4.4) do not bear tildes: independent and proportional costs both matter for purchaser-price elasticities, though only independent ones matter for price-ratio elasticities. Note also that country  $z$ 's average share of any market for any pair of goods depends importantly on other things, too, including its production costs and its size: larger countries tend to have larger shares of all markets.

That trade costs put home suppliers at an advantage in home markets helps to explain why empirical studies consistently find home bias in patterns of consumption (Trefler, 1995; Davis and Weinstein, 2001). In the present model, home bias arises also from the variation of relative factor prices among countries with their endowments: more is consumed of goods which a country can produce more cheaply (Romalis, 2004: 70).

Pulling the preceding discussion together, the producer-price elasticity of demand for a country's exports is usually similar in form to that of earlier sections

$$\varepsilon_{j1}^{z*} = \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}^{Dz*} + \tilde{\tau}_{j1}^{Fz*}} \quad (4.5)$$

because export market shares are usually small. The only change is the expansion of the denominator to separate the trade costs that country  $z$  incurs in sending goods to its export markets from the trade costs that it incurs within those markets (which are usually similar for all suppliers). Equation (4.5) refers to total exports, but could be modified to refer to any individual export market by substituting  $\check{z}$  ( $\neq z$ ) for  $*$ . The distribution of a country's exports across individual markets depends on the relative sizes and trade costs of the markets, in ways that are analysed in gravity models.

There is more of a difference from earlier sections in the producer-price elasticity of demand for home market sales, which is approximately

$$\varepsilon_{j1}^{zz} = \frac{\beta_{j1} - s_{j1}^{zz}(t_{j1}^{F*z})(\beta_{j1} - \gamma_{j1})}{1 + \tilde{\tau}_{j1}^{Dzz}} \quad s_{j1}^{zz}(\ ) > 0 \quad (4.6)$$

The purchaser-price elasticity in the numerator is smaller than for exports, to an extent that depends on how much smaller  $\gamma_{j1}$  is than  $\beta_{j1}$  and on the degree of protection of the home market for these goods by the international trade costs of foreign suppliers. By contrast, the price-ratio elasticity for home sales is usually higher than for exports, because the denominator of (4.6) includes only internal trade costs. The producer-price elasticity for home market sales could thus in principle be either lower or higher than for exports. In the limiting case of a closed economy or a pair of non-traded goods, the home market producer-price elasticity is

$$\varepsilon_{j1}^{zz} = \frac{\gamma_{j1}}{1 + \tilde{\tau}_{j1}^{Dzz}} \quad (4.7)$$

which is lower than for home market sales of otherwise similar traded goods (and is usually, though not necessarily, lower than for exports of such goods).

The producer-price elasticity of demand for a country's total outputs of goods  $j$  and 1, which must lie between  $\varepsilon_{j1}^{z*}$  and  $\varepsilon_{j1}^{zz}$ , is thus

$$\varepsilon_{j1}^{\check{z}} = \frac{\beta_{j1} - s_{j1}^{\check{z}}(\beta_{j1} - \gamma_{j1})}{1 + \tilde{\tau}_{j1}^{\check{z}}} \quad (4.8)$$

where  $s_{j1}^{\check{z}}$  and  $\tilde{\tau}_{j1}^{\check{z}}$  are averages across all the country's markets, abroad and at home, weighted by its sales in each of them. The share in each market is affected by trade costs, as in equation (4.4), while the weight depends also on the size of that market.<sup>10</sup> Higher independent trade costs tend to lower  $\varepsilon_{j1}^{\check{z}}$  by raising  $\tilde{\tau}_{j1}^{\check{z}}$ . Higher international trade costs, both independent and proportional, tend to raise  $s_{j1}^{\check{z}}$ , but also to reduce  $\tilde{\tau}_{j1}^{\check{z}}$  by raising the share of output sold in the home market, where the country's producers incur only internal trade costs, so their net effect on  $\varepsilon_{j1}^{\check{z}}$  could be in either direction.

<sup>10</sup> Algebraically,  $s_{j1}^{\check{z}} = \sum_{\check{z}} (s_{j1}^{\check{z}})^2 \mu_{j1}^{\check{z}} / \sum_{\check{z}} s_{j1}^{\check{z}} \mu_{j1}^{\check{z}}$ , where  $\mu^{\check{z}}$  is the overall size of market  $\check{z}$ .

The producer-price elasticity for imports can be written approximately as<sup>11</sup>

$$\varepsilon_{j1}^{*z} = -\frac{s_{j1}^{zz}(t_{j1}^{F*z}) (\beta_{j1} - \gamma_{j1})}{1 + \tilde{\tau}_{j1}^{Dzz}} \quad (4.9)$$

which shows how changes in the relative producer prices of a country's varieties of goods  $j$  and 1 affect the relative sales in its home market of these goods by foreign suppliers. The sign of (4.9) is negative, unlike the other producer-price elasticities, because changes in a country's prices push its own sales and those of its competitors in opposite directions. Its denominator is the same as for the home sales elasticity (4.6), since what matters for imports is how changes in the country's producer prices affect its purchaser prices in the home market. The purchaser-price elasticity in the numerator, and hence the whole equation, is undefined for pairs of goods of which a country has either no sales in its own market (non-competing imports) or supplies the whole of it (non-traded goods). Within these limits, the effect of a country's prices on the sales of its competitors decreases with its share of its home market.

#### 4.2 Multi-market H-O relationships

The market-specific producer price elasticities defined above can be used as needed to replace those in the versions of the model set out in sections 2 and 3, which assumed small market shares for all goods. Their use is illustrated below with the simplified version of the higher-dimensional model.

By combining equations (4.5) and (4.9) with (3.10), the influence of a country's factor endowments on the composition of its trade can be written for exports as a set of equations of the form

$$\hat{q}_j^{z*} - \hat{q}_1^{z*} = \frac{\beta_{j1}}{1 + \tilde{\tau}_{j1}^{Dz*} + \tilde{\tau}_{j1}^{Fz*}} \sum_{i=2}^m (\theta_{ij} - \theta_{i1}) \rho_{i1}^z (\hat{v}_i^z - \hat{v}_1^z) \quad (4.10)$$

and for imports as a set of equations of the form

$$\hat{q}_j^{*z} - \hat{q}_1^{*z} = -\frac{s_{j1}^{zz}(t_{j1}^{F*z}) (\beta_{j1} - \gamma_{j1})}{1 + \tilde{\tau}_{j1}^{Dzz}} \sum_{i=2}^m (\theta_{ij} - \theta_{i1}) \rho_{i1}^z (\hat{v}_i^z - \hat{v}_1^z) \quad (4.11)$$

As usual in a H-O model, a larger relative endowment of a factor tends to increase the relative exports, and decrease the relative imports, of goods in whose production it is used relatively intensively. For any given pair of goods, the effect of endowments on relative exports could be either larger or smaller absolutely than the effect on relative imports, depending on how trade costs affect producer-price elasticities. International trade costs push them in opposite directions: for exports, the producer-price elasticity is reduced by (independent) international trade costs, while for imports, it is increased by international trade costs (proportional as well as independent).

<sup>11</sup> Rewrite equation (4.3) as a relationship in the home market between foreign sales and foreign prices, denoting the (foreign) market share by  $s^*$ . Convert this into a relationship between foreign sales and the prices of the home country by subtracting  $\gamma_{j1}$ , for reasons explained in connection with equation (2.12), rewrite  $s^*$  as  $(1 - s)$ , simplify, and add a negative sign to define it like a substitution elasticity.

Trade costs also push (4.10) and (4.11) in the same direction, through their effect on the shared  $\varphi_{i1}^z$  term. In both cases,

$$\varphi_{i1}^z = \frac{1}{\sum_{j=1}^n (\lambda_{ij} \sigma_{iji} - \lambda_{1j} \sigma_{1ji}) + \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j}) \varepsilon_{j1}^z (\theta_{ij} - \theta_{i1})} \quad (4.12)$$

which differs from equation (3.9) in replacing  $\beta_{j1}/(1 + \tilde{\tau}_{j1})$  in the second denominator term with  $\varepsilon_{j1}^z$ , the producer-price elasticity for the total outputs of each pair of goods from (4.8). The summations in (4.12) are also over all of the country's goods, while the trade costs shown in (4.10) and (4.11) are just those of the goods concerned. Thus since trade costs usually reduce  $\varepsilon_{j1}^z$ s, a generally high level of trade costs in a country, by increasing the impact of endowments on factor prices, tends to amplify the effects of endowments on the composition of both exports and imports.

The effects of trade costs on market shares also condition the influence of changes in foreign prices on factor prices, which were analysed earlier on the assumption that the country had only small market shares. Substitution of (4.3) into (2.12) yields

$$\hat{q}_j^{zz} - \hat{q}_1^{zz} = -[\beta_{j1} - s_{j1}^{zz} (\beta_{j1} - \gamma_{j1})] (\hat{p}_j^{zz} - \hat{p}_1^{zz}) + (1 - s_{j1}^{zz}) (\beta_{j1} - \gamma_{j1}) (\hat{p}_j^{*zz} - \hat{p}_1^{*zz}) \quad (4.13)$$

which describes the effects of foreign prices on country  $z$ 's sales in one market. Since what matters for the country's factor prices is the average change in foreign purchaser prices across all its markets (whether caused by changes in foreign trade costs in those markets or by a global change in foreign producer prices), equations (3.13) become

$$\hat{w}_i - \hat{w}_1 = \varphi_{i1}^z \sum_{j=2}^n (\lambda_{ij} - \lambda_{1j}) (1 - s_{j1}^z) (\beta_{j1} - \gamma_{j1}) (\hat{p}_j^* - \hat{p}_1^*) \quad (4.14)$$

where, as in (4.8),  $s_{j1}^z$ ,  $p_j^*$  and  $p_1^*$  are weighted averages across all markets, while  $\varphi_{i1}^z$  is determined by (4.12). A generally high level of trade costs across all goods, by raising  $\varphi_{i1}^z$ , amplifies the effect of a given-sized change in factor demand on factor prices. However, high trade costs on any individual pair of goods,  $j$  and 1, by raising  $s_{j1}^z$ , reduce the change in factor demand that results from a given-sized change in their foreign prices – and for non-traded goods, with  $s_{j1}^z = 1$ , equation (4.14) confirms that changes in their foreign prices have no effect on factor prices.

In the multi-market version of the present model, as in the versions in sections 2 and 3, H-O forces operate in familiar directions. For example, a greater endowment of a factor tends to raise the exports of goods that are intensive in that factor, and a fall in the foreign price of a good tends to lower the prices of factors used intensively by that good. In all versions of the model, too, factor prices are affected by endowments, unlike the one-cone HOS and HOV models.

What varies among these versions, and what distinguishes them from standard H-O models, are the ways in which the sizes of H-O effects depend on trade costs. Simple statements about how trade costs condition these effects become harder once one gets beyond the small-country 2x2 model of section 2. One cannot, for example, make general statements of the form ‘trade costs reduce the responsiveness of relative outputs to relative factor endowments’. This is because there are many goods in an economy, each with its own trade costs and often sold in several markets, and because there are different sorts of trade costs – independent and proportional, international and internal, and those of foreigners and of local suppliers.

For instance, equations (4.10) and (4.12) show that the net effect of a country having generally high trade costs on the responsiveness of the composition of its exports to its endowments is ambiguous. By reducing the output elasticities in (4.8), and by making more goods and services non-traded (with  $\varepsilon_{ji}^z$ s that are normally lower than for traded goods, as in (4.7)), generally high barriers to trade increase the sensitivity of factor prices to endowments. Pulling the other way, though, high barriers to trade reduce the responsiveness of exports to factor prices (4.10). Similarly, equations (4.12) and (4.14) between them show that the effects of changes in foreign prices on factor prices could be made either larger or smaller by higher trade costs.

Useful and accurate statements about the effects of trade costs on these and other H-O relationships can still be made, but they need to be more precisely formulated – what sorts of trade costs, in what markets? For example, if the independent international trade costs of local suppliers of a subset of goods were higher, the responsiveness of the relative exports of those goods to the relative endowments of the country would be lower. Or if a subset of goods in an economy became non-traded as a result of high international trade costs (both independent and proportional) on foreign and local suppliers, the responsiveness to endowments of the relative exports of other goods would be greater. There is much scope for further analysis of this sort.

## **5. Issues in empirical application**

The model in this paper embodies the two basic insights of Heckscher and Ohlin: that the composition of a country’s trade is influenced by its factor endowments, and that a country’s factor prices are influenced by trade. The model owes much to the brilliant formalisation of H-O by Samuelson, but is more consistent with empirical evidence, including the three findings of Davis and Weinstein (2001): relative factor prices vary with endowments; efficiency varies across countries in an approximately sectorally neutral way; and the composition of consumption has a home bias.

Like any theoretical model, however, the one set out above is a simplified abstraction, which still makes many unrealistic assumptions. This final section considers whether, despite these assumptions, the model is useful for analysing the real world, and how it should (and should not) be used in empirical work.

### *5.1 Violation of assumptions*

The algebra of the model refers to small changes. Application requires its extension to big changes, including big differences across countries, especially in endowments. Formally, this is easy to do by working in logarithms – replacing hatted variables with

log differences, so that  $\hat{v}_i - \hat{v}_1$ , for example, becomes  $\Delta \ln(v_i/v_1)$ , which can be of any size – but it requires assumptions that cannot be strictly correct. Key elements of the model that can be treated as parameters with small changes, especially the  $(\theta_{ij} - \theta_{1j})$ ,  $(\lambda_{ij} - \lambda_{i1})$  and  $\tilde{\tau}_{j1}$  terms, are actually endogenous. Insofar as the world is not CES, the substitution elasticities  $(\sigma_{iji}, \beta_{j1}, \gamma_{j1})$  will also change. And there is scope for variation of all the parameters among countries.

Violations of other more common H-O assumptions, too, must be remembered. Non-neutral or Ricardian differences in the technologies of some sectors may be important for some countries (Harrigan, 1997). Assuming producer prices to be tightly linked to production costs by competition is not sensible for some goods in some countries, nor during adjustments to large shocks. The assumption that all countries have the same preferences is at best an approximation, and deviations from homotheticity may need to be allowed for in studies across all countries in the world. As in other H-O models, factor intensity non-reversal is vital: even if the  $(\theta_{ij} - \theta_{i1})$  and  $(\lambda_{ij} - \lambda_{1j})$  terms vary in size somewhat with endowments, they must retain the same signs.

For all these reasons, the model can be only approximately accurate, but it seems to work well by the usual standards of applied economics. Variation across countries in logged endowment ratios explains a considerable proportion of the variation in logged export ratios (of pairs of goods), with the signs of the coefficients on the endowment ratios being consistent with other evidence on the factor intensities of the goods. An early study was Keesing and Sherk (1971), with more recent applications in papers by myself and co-authors.<sup>12</sup> The main focus has been on the composition of exports, but there has been some analysis of outputs and of imports. As in Leamer (1984), it seems possible to estimate the effects of more than two factors on sectoral structure, despite the theoretical possibility of not being able to do so. There is scope for more work of this kind, especially on the effects of trade costs, and for links with gravity models to extend it to the directions as well as the commodity composition of trade.

The strongest assumption of the model in its use for explanation of trade and output structures is constant returns to scale. Increasing returns are enormously important in manufacturing and services, as emphasised in models with monopolistic competition and in economic geography, while in primary sectors, there may be decreasing returns because of variation in the quality of natural resources within countries. H-O models are thus more useful in some domains than in others. They are unable to explain trade among countries with similar endowments, or the details of the composition of trade. Even where H-O models are most relevant – namely to broad patterns of trade among countries with different endowments – they may be made more accurate by allowing for scale economies (Keesing and Sherk, 1971; Wood and Mayer, 2001).

The usefulness of H-O theory in explaining changes in factor prices, and particularly the effects of falling trade costs on wages, has been much debated (*e.g.* Wood, 1998; Anderson, 2005; Baldwin, 2008, ch. 5). The present model, however, seems likely to be of more use for this purpose than other H-O models. The basic difference is that in the present model wages do not depend simply on foreign goods prices, as in a one-

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<sup>12</sup> Including Wood and Berge (1997), Owens and Wood (1997) and Mayer and Wood (2001). Wood (2003: 169-78) and Wood (2009) summarise these and other papers. The specification used in these papers is tested against other specifications by Aldaz-Carroll (2003).



cone HOS model, but are determined by supply and demand in the labour market, of which foreign goods prices affect the demand side. The volume of trade also matters for wages, so that factor content calculations are a better way of estimating the effects of trade than in a HOS model.<sup>13</sup> Non-traded sectors, too, play more of a role. And since factor prices vary with endowments, it makes sense to control for changes in endowments when estimating the effects of changes in trade (Robbins, 1996).

That these features of the present model make it of more practical use for explaining how trade affects wages is an assertion, based on my interpretation of the evidence in this field. Even if this assertion is correct, however, it is unlikely that any H-O model could provide a complete explanation of the effects of trade on wages. In many cases, it is necessary to allow also for trade-induced changes in technology (and to bear in mind that in the present model, unlike the HOS model, relative wages can be affected by factor-biased as well as sector-biased technical change). Examples are defensive innovation in developed countries (Wood, 1994) and skill-biased technology transfer in developing countries (Robbins, 1996). Explaining the effects of trade on wages in recent decades in both developed and developing countries seems to require a mixture of H-O and other theories (Wood, 2002; Anderson et al, 2006).

## 5.2 Aggregation of goods and factors

The choice of goods and factors to be used in empirical applications is not constrained by the present model, which works in any dimensions, but depends on the purpose of the analysis and the availability of data. An important issue with both goods and factors, however, is aggregation.

‘Goods’ are usually value aggregates of many goods, as Davis and Weinstein (2001) note. Variation of factor intensity among the goods within each aggregate makes H-O relationships less precise. In addition, the average factor intensity of the goods within a given statistical category varies among countries (and over time) with endowments, for the same reasons and in the same ways that the broad structure of output varies in H-O theory. Especially when factor prices vary with endowments, as in the present model, a country with more of factor  $i$  not only has larger  $i$ -intensive sectors, but also, in all sectors, has more  $i$ -intensive mixtures of goods (as shown by Schott, 2003).

As a result, the measured composition of output varies less with endowments than it would if goods were homogeneous. Statistically, the effect is indistinguishable from that of the use in a more  $i$ -abundant country of more  $i$ -intensive techniques to produce all goods. But its economic meaning is different, since different countries are actually producing different goods – in terms both of their mixture and of their existence. The assumption of the present model that there are national varieties of all goods, and thus that all countries produce some of all goods at all levels of disaggregation, is just a convenient fiction. In detailed reality, no country produces all goods.

Aggregation of goods does not affect the relationships in the present model between endowments and factor prices. As can be seen, for example, from equation (4.12), these economy-wide relationships involve adding more detailed relationships over all

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<sup>13</sup> Factor content calculations are debated in JIE (2000). Even in the present model, it is non-competing imports that pose the biggest challenge for such calculations (Wood, 1994: 72-4).

pairs of goods, which can be supposed to be fully disaggregated. There is no practical need for the goods categories used in these relationships to be the same as those used in applying the model to explain, say, the composition of exports.

Aggregation also matters for factors, which are bound to be heterogeneous in type and quality. H-O models depend on the mobility of factors among sectors, which creates a presumption that factors should be defined and measured in a way that makes each factor usable in all sectors. Sector-specific factors are important, however: in theory, in empirical explanation of the detailed composition of trade, and in considering the distributional effects of trade policy changes. In practice, it is thus often useful to blend elements of a specific-factors model into a H-O model.

Even abstracting from sector-specificity, however, it is necessary to decide how finely factors should be divided. For example, should human capital be treated as a single factor, or should workers be split into two or three skill categories, or should they be split into a larger number of years-of-schooling groups? For some purposes, a finer disaggregation may be better, but this tends to increase the degree of substitutability among factors and thus to reduce the accuracy of the simplified version of the present model in higher dimensions, for reasons explained at the end of section 3. The model is likely to work best empirically with a few broadly defined factors that are neither close substitutes nor strong complements for one another.

A further issue is whether (non-human) capital should be one of these broad factors, as it has been in most empirical H-O studies. Capital is often strongly complementary to skill (or human capital), which could add to the inaccuracy problem just mentioned. A more fundamental challenge to the appropriateness of including capital, however, is the high degree of international mobility both of capital goods and of financial capital. As shown by Ethier and Svensson (1986), the factors relevant to a H-O explanation of trade in goods are only those that are internationally immobile. The way in which endowments of capital are conventionally measured is also open to both theoretical and empirical challenge (Wood, 1994, section 2.2).

Whether capital should be included may vary with the purpose and scope of the study. In my own empirical work, which has been concerned mainly with broad variations across the world as a whole, I have omitted capital and focused on endowments only of labour, skill and land. If capital is mobile, other H-O studies with similarly broad scope that obtain reasonable results with capital included may be capturing the effects of skill, with which capital is strongly correlated across both countries and goods. But for some purposes one should recognise that the prices even of traded capital goods vary somewhat among countries, and that interest rates in some countries may diverge persistently from world levels because of barriers to capital flows or country risk.

In conclusion, the H-O model in this paper seems useful for analysing the real world, although, as in all empirical work, judgement is needed in using it, including on when and how to combine it with elements of non-H-O theories. The model can surely be improved, but the main scope for further work is in applying it empirically. Perhaps the biggest opportunity is for better analysis of the effects of trade costs, with more of a distinction than in earlier research between independent and proportional costs.

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