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Hurdle Rate Mistakes Reconsidered

Adrian Wood[1]

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Abstract

The hurdle rates used by firms in making investment decisions are typically higher than their costs of capital. Hurdle rates also respond to changes in the cost of capital only very slowly. A non-standard theoretical explanation of these facts, set out in this paper with a simple model, is that firms are successfully and inconspicuously managing persistent quantity constraints on access to finance. The policy implications of this explanation conflict with those of the standard 'managerial mistakes' explanation.

[1] University of Oxford, Department of International Development (email: adrian.wood@qeh.ox.ac.uk). The theory in this paper owes much to Nicholas Kaldor and Robin Marris, as explained in Wood (2025), which also discusses related earlier work by others. I am indebted to Thomas Dallery, Valpy Fitzgerald, Joel Rabinovich, and Engelbert Stockhammer for help in improving the model, to Kilian Huber for supplementary results, and to Huber and Rabinovich for comments on a draft.

That a firm's investment level depends inversely on some measure of its cost of capital related to the prevailing interest rate is a key assumption of much economic theory, micro and macro. However, the hurdle rates that firms use in investment decisions have often been observed to be much higher than their costs of capital.[2]

In outstanding empirical research, Niels Gormsen and Kilian Huber (2025) scrape information on hurdle rates and costs of capital from the conference calls with investors of about 1300 corporate firms in fifteen developed countries over two decades.[3] They use these data, which are unfortunately inaccessible in their raw form to other researchers for legal reasons, to make two important contributions to our understanding of this issue.

The first is to show how much the observed 'wedges' between hurdle rates and costs of capital depend on the treatment of overhead costs. Textbooks recommend that firms set hurdle rates as if project-level calculations of expected returns on investment will include an allowance for overhead costs. Only about 15% of the Gormsen and Huber (G&H) hurdle rate observations, however, are from firms that follow this advice. In the great majority of firms, overhead costs are covered instead by setting hurdle rates well above minimum required (and realised) rates of return. The average reported hurdle rate for all G&H firms is 4.2 percentage points above the 11.5% average for the textbook-compliant firms (G&H Table 1).

The second G&H contribution, which is the main analytical focus of their paper and exploits the exceptional time-series dimension of their data, is to show that changes over time in the cost of capital cause changes in hurdle rates (which inversely affect investment). This result is potentially relevant for macro analysis and monetary policy, but with the crucial qualification that adjustment is very slow. A one-percentage-point rise in a firm's cost of capital typically has almost no effect for a couple of years and its full effect – on average a 0.7-point rise in the firm's hurdle rate – is attained only after twelve years (G&H Figure 4).

The size of the hurdle rate response to a change in the cost of capital also varies widely across firms. The 90% confidence interval around the eventual 0.7 point estimated average adjustment is 0.8 points. Moreover, although falling interest rates over the G&H data period (2002-2021) reduced the average cost of capital by about 3 percentage points, there was no downward trend in the average hurdle rate, so the average wedge rose by about 3 points (G&H Figure 2).

G&H undertake less analysis of variation in hurdle rates and wedges across firms. It is clear, however, that wedges are common and vary substantially among firms. For all firms, the mean hurdle rate is 7 percentage points above the mean perceived cost of capital and a regression of hurdle rates on perceived costs of capital explains only 17% of their variation (G&H Table 1 and Table 4, column 1). For the small minority of firms that follow textbook advice, with no

[2] Earlier empirical literature on hurdle rates is briefly surveyed by Gormsen and Huber (2025, pp 2006-7).
[3] G&H call hurdle rates 'discount rates', though they see these two labels as equivalent. 'Hurdle rate' is used here because this label conveys its use more directly. This label is also a better match with the usual way that firms use discounted cash flow methods to assess investment projects, which is by calculating their internal rates of return, despite the advice of corporate finance texts to calculate net present values (Brealey et al., 2023, ch. 5).

overhead adjustment, the 5th and 10th percentiles of the wedge are zero, the median wedge is 2 percentage points, the 90th percentile is 4 points and the 95th percentile is 5 points.[4]

In conference calls, the managers of firms with wedges provide a variety of plausible-sounding explanations for their non-compliance with the value-maximising principle of setting hurdle rates equal to costs of capital. G&H, however, argue that only two of these explanations make economic sense. One is firm-specific risks that are known to firms but not to financial markets. The other is firms – especially small or new ones – running short of cash through errors or bad luck. All the other explanations – for example that higher hurdle rates reassure investors that their money is being used in a disciplined way – are simply economic mistakes, which the firms concerned are able to survive only because they have monopoly power in markets.

G&H test econometrically their three reasons for wedges – the two rational explanations and monopoly power. All are statistically significant but explain little of the variation among firms in the speed of their hurdle rate responses to changes in costs of capital (G&H Tables 6 and 7). Even when combined, moreover, they account for less than a tenth of cross-firm variation in the size of wedges (G&H Table A10). The monopoly power reason is also questionable: the evidence for it is a positive statistical association between wedges and profit margins, but the profitable firms driving this association might be high-productivity superstars (Autor et al., 2020) rather than inept monopolists.

In summary, this pathbreaking G&H work generates as many questions as answers – including questions about the standard theory of corporate finance. Two features of the evidence, though, hint that wedges might be enduring and desirable – rather than as in standard theory temporary or pathological – and also hint at why adjustment is so slow. One feature is that more profitable superstar firms also grow faster (e.g., Baqaee and Farhi, 2020). The other is that the estimated hurdle rate adjustment lag structure resembles the term structure of corporate bonds.

Both features, in conjunction with the widespread existence of wedges, suggest that many firms – not just small or new ones, and particularly growing ones – are subject to quantity constraints on their access to finance. The wedges are thus closely related to the Lagrange multipliers of these firms' constrained optimisation decisions. Wedges vary in size among constrained firms because of variation in the balance between their investment opportunities and their access to finance. Slow adjustment to changes in the cost of capital in constrained firms reflects the need to await the maturation of outstanding long-term debt commitments.

The quantity constraints on finance that cause wedges differ from the 'distress' constraints on the small share of firms that are in financial difficulties (only 7% in Hadlock and Pierce, 2010).[5] They are 'strategic' constraints which are anticipated by firms, built into their business plans, and managed in ways that avoid them derailing these plans. Nor is this distinction between 'distress' and 'strategic' constraints the one made in corporate finance texts between 'hard' and 'soft' quantity constraints, with the soft constraints being created within firms by restriction of departmental investment budgets (e.g., Brealey et al., 2023, pp. 137-8). 'Strategic' constraints are hard constraints on the firm as a whole.

[4] These statistics on textbook-compliant firms were kindly provided by Kilian Huber.

[5] Adding together the 'financially constrained' and 'likely financially constrained' groups in their Table 1.

Strategic quantity constraints exist because in almost every firm there is an upper limit within its planning horizon on the available amount of external and internal finance – set by leverage risks, by the resistance of shareholders to new issues, by the attainable level of profits, and by the need for part of the profits to be distributed to shareholders. For some firms this limit is irrelevant because it lies beyond the sum of their potentially profitable investment projects. But for firms with better investment opportunities, even if as a result they also have better access to finance, this limit can become a binding constraint.

If such firms are well managed they need never run into short-term financial difficulties, even though they cannot realise their full investment potential. They recognise that the textbook rule of investing in everything that offers a rate of return above their cost of capital would lead to financial disaster. What prevents disaster is the wedges in their hurdle rates – though it could alarm investors to explain wedges that way, which may well be why only 3% of firms disclose their hurdle rates (G&H, pp. 2018-19). Unpredictable short-term variations in revenues and costs are managed by varying stocks of liquid financial assets and short-term debt.

The realistic assumption that a typical firm is well-managed in this short-term financial sense enables the theoretical explanation of wedges outlined above to be embodied in a simple model of long-term firm growth subject to a quantitative financial constraint. The hurdle rate of an unconstrained firm is its exogenous cost of capital, with no wedge. In a constrained firm, the (higher) hurdle rate is endogenous: it is the hypothetical cost of capital that would cause an optimising firm to choose the quantitatively constrained level of investment.

Unlike most of the large literature on financial constraints on investment (e.g., Hubbard, 1998; Christodoulou et al., 2022), the key effect of financial market imperfections in this paper's model is on the quantity of finance that a firm can access rather than on the spread between its costs of internal and external finance. A higher cost of external capital raises a quantitatively unconstrained firm's hurdle rate but does not create a wedge. In a quantitatively constrained firm, a higher cost of external capital could raise the hurdle rate by lowering the amount of external finance that the firm chose to use, for example by lowering its target leverage ratio, but its hurdle rate is generally above this external cost.

The hurdle rate in a quantitatively constrained firm is the internal opportunity cost of retained earnings – the return forgone at the margin by not investing retained earnings in physical assets. Its wedge is the difference between this internal opportunity cost and the external opportunity cost of retained earnings – the return forgone by not investing in financial assets of similar risk – minus the spread added to its cost of capital by financial market imperfections. The effect of a higher cost of capital on a constrained firm's wedge is ambiguous, because it could also raise the hurdle rate, but is likely in most cases to be a reduction of the wedge.

From a policy perspective, it matters greatly whether most hurdle rates being higher than costs of capital is caused mainly, as G&H suggest, by managerial mistakes or, as argued in this paper, by strategic financial constraints. If G&H were right, investor action or government regulation to force closer alignment of hurdle rates with costs of capital would cause beneficial increases in investment, whereas, if the present paper were right, such interventions would inflict serious economic damage.

There is also an income distribution policy dimension. Strategic financial constraints would not only cause hurdle rates to exceed costs of capital but would also raise the profit margins of the firms concerned and the share of profits in national income. The implications for policy are not straightforward (Wood, 1975 and 2025): insofar as the higher profits were compensating for a financial market failure there might be no efficiency case for intervention, but if they also enriched shareholders there would be an equity case for carefully targeted taxation.

In section 1 below, familiar long-term quantity limits on access to finance are combined into a firm's 'finance frontier', which sets a lower bound on the relationship between its profit margin and its growth rate. This frontier is juxtaposed with the firm's opportunity frontier (which is an upper bound on this relationship) in a diagram that defines whether it is financially constrained. Section 2 uses tractable forms of these frontiers to specify more precisely where the financial constraint binds and the levels of the hurdle rate and the wedge in a constrained firm. Section 3 considers the consistency of this theoretical framework with the G&H results in more detail, within limits set by the inaccessibility of their data, and Section 4 concludes.

1. Finance frontiers and opportunity frontiers

That wedges in hurdle rates reflect strategic quantitative constraints on firms' access to finance was implicit in Wood (1975, pp. 97-100, 119-21), whose theory is developed further here.[6] It integrates finance into a model of a growing firm (*ibid.*, sections 3.4-6) that connects decisions about investment in capacity expansion with decisions about market expansion and especially about profit margins (which affect the availability of internal finance).

These are long-term decisions. Firms are assumed to have rolling business plans, including hurdle rates, which are adjusted periodically in response to experience (although only a single long period is modelled). Alongside their long-term plans, firms have contingency plans for unpredictable short-term fluctuations of revenues and operating costs, for which they maintain and use liquidity cushions of financial assets and short-term borrowing capacity.[7] This short-term financial management is assumed to be successful and is not formally modelled.

Chapter 2 of Wood (1975) explains how ignorance and differing views of the future, combined with information asymmetries, lack of trust, and transactions costs create (what are called here) strategic quantitative limits on access to both external and internal finance. The following four paragraphs summarise its explanation, which with minor updates is broadly consistent with the relevant parts of current finance textbooks (e.g., Brealey et al., 2023, Chapters 13-18).

[6] This 1975 theory was ahead of its time in rejecting the then orthodox view that a firm could finance externally any investment that it would be willing to finance out of retained profits (p. 6), but like some other theories of the firm at that time it assumed that managers sought to maximise growth rather than shareholder value. Although the book was described as 'required reading' in *The Journal of Finance* (Krouse, 1977) and the theory is used in post-Keynesian economics (Wood, 2025), these two features of the theory in combination probably explain its neglect in the revolution over the next couple of decades in mainstream thinking about corporate finance.
[7] The fundamental theoretical reasons for which are dissected in Chen, Giroud, Qin and Wang (2026).

The share of investment that a firm is willing to finance by long-term borrowing is limited by its target balance sheet ratio of debt to equity. This target reflects the risks that high leverage ratios create for a firm's freedom of action and the well-being of its shareholders. The chosen ratio varies among firms with the inherent volatility of their profits, the risk-aversion of their shareholders, and the potential terms available for borrowing.[8] The chosen ratios may also vary over time with the general level of interest rates.

Physical investment by an established company is only occasionally and exceptionally financed by raising new equity capital. Share issues are disliked by shareholders because they usually drive down the share price – the presumption being that successful firms should return money to their shareholders, not ask them for more. If the market is impressed by the extra investment – physical or in an acquisition – that a share issue would permit, the share price could rise, but the information effects of new issues are hard to predict.

By far the biggest source of finance for investment in most established firms is their retained earnings.[9] The main determinant of its availability is the size of the firm's profits (defined here as EBITDA – earnings before interest, tax, depreciation and amortisation). Interest payments depend on a firm's leverage and on the average interest rate on its debt.[10] Tax payments depend on leverage and tax rates. Retained earnings then depend on the share of profits accounted for by depreciation and on the share of net profits paid out as dividends and share repurchases.

In principle, a firm pursuing the interests of its shareholders should pay back to them whatever net profits it could not invest at a rate of return at least equal to what they could earn from other investments of equal risk. In practice, shareholders dislike dividend volatility, partly because rises and falls have asymmetric information effects on share prices. So companies tend to aim for stable and steadily rising dividends, which in any given planning period and regardless of the logic behind earlier dividend decisions sets a lower limit on 'normal' payouts, with share repurchases providing upward flexibility. An upper limit on internal finance is what retained earnings would be if nothing was distributed to shareholders (as in most new firms).

These limits on the available quantities of different sorts of finance, which have been modelled individually in other recent studies,[11] can be combined into a finance frontier. This frontier is a lower bound on the long-term relationship between two variables that are central to the long-term plan of any non-financial firm: the growth rate of its sales revenue, denoted by g, and the average margin of profit in its sales revenue, denoted by π. The finance frontier can be reduced (Wood, 1975, section 3.3) to a simple inequality:

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[8] Since Wood (1975), supply-side limits on borrowing created by the need for collateral (e.g., Stiglitz and Weiss, 1981) have been eased by the emergence of junk bonds and private credit at higher interest rates.

[9] The composition of company finance by broadly defined source remains much the same as at the time of Wood (1975, p. 9), except for the growth of share buybacks as a supplement to dividends: Bank of England (2016, Charts 14, 25), Brealey at al. (2023, Figure 13.1).

[10] Offset by interest received on its cushion of financial assets, though by convention this interest is omitted from the standard EBITDA measure of the performance of non-financial companies.

[11] For example, the effects of limits on borrowing are modelled by Tomat (2008) and the effects of lack of access to external finance by Moyen (2004). Wroblewski (2025) assumes lack of access to external finance plus a lower limit on dividends in a study of the short-term response of investment to changes in the interest rate.

$$(1) \qquad \pi \geq \frac{1+f-x}{e} kg$$

The financial asset ratio, f, is the firm's addition to its desired cushion of financial assets relative to its investment in physical assets. The external finance ratio, x, is the maximum share of its physical investment that can be financed by borrowing or share issues. The earnings retention ratio, e, is the maximum proportion of profits that can be retained for investment after paying interest, taxes and the minimum dividends and share repurchases that shareholders will accept. The investment coefficient, k, is the ratio of gross physical investment to the resulting increase in the firm's output capacity (assumed to be equal in the long term to its sales).[12]

Equation (1) links the long-term level of a firm's profits, P, to the long-term level of its physical investment, I, with both variables normalised on sales revenue, R. It says that the ratio of profits to sales revenue, π, must be no less than some multiple of the investment to sales ratio, gk (since $g = \Delta R/R$ and $k = I/\Delta R$), with the multiple governed by the financial variables f, x and e.

If $(1 + f - x)$ is less than unity, as has historically been the case for most firms, with additions to financial cushions being smaller than external finance for physical investment, it measures the share of physical investment that must be financed from retained earnings. The increased importance of intangible assets, though, has reduced physical investment and raised f (to cover urgent unexpected replacement of intangibles).[13] So $(1 + f - x)$ has become greater than unity for some companies, whose need for retained earnings exceeds their physical investment.

Whatever the value of $(1 + f - x)$, its multiplication by gk determines a company's need for retained earnings relative to its sales. Division of this need by e, which is the maximum ratio of retained earnings to profits, determines the minimum necessary amount of profits relative to sales, with a lower e requiring a higher minimum π. Equation (1) as a whole thus specifies the minimum profit margin needed for a given growth rate to be financially sustainable.

To complete the model, an opportunity frontier

$$(2) \qquad \pi \leq \mu(g,k) \qquad \mu_g^{'} < 0,\ \mu_k^{'} > 0$$

(where μ is *mu* for 'margin') summarises the investment opportunities of a firm as an inverse relationship between its maximum attainable profit margin and the long-term growth rate of its sales revenue. The maximum attainable level of a firm's profit margin at any given growth rate rises with its investment coefficient, k, because replacing old equipment can lower the cost or improve the quality of its products.

[12] G&H (2025) use k to refer to total capital stock. Other notation in this paper is consistent with G&H, but differs from Wood (1975, 2025) by using e for the retention ratio rather than r (used by G&H for the cost of capital).
[13] Spending on intangible knowledge-based assets is classified mainly as operating costs. Such assets are at risk of sudden obsolescence from new knowledge or competition, requiring rapid replacement, which must be financed from reserves because it generates no collateral for borrowing (Chen *et al.*, 2017; Li, 2025).

A firm's best attainable profit margin is inversely related to its growth rate mainly because of competition with other firms: to achieve faster growth of sales requires lower prices and higher expenditure on marketing and product improvement, which tend to reduce profits. This inverse relationship also reflects the 'adjustment costs' associated with the higher levels of investment required for faster growth of output (e.g., Hubbard, 1998, p. 202).

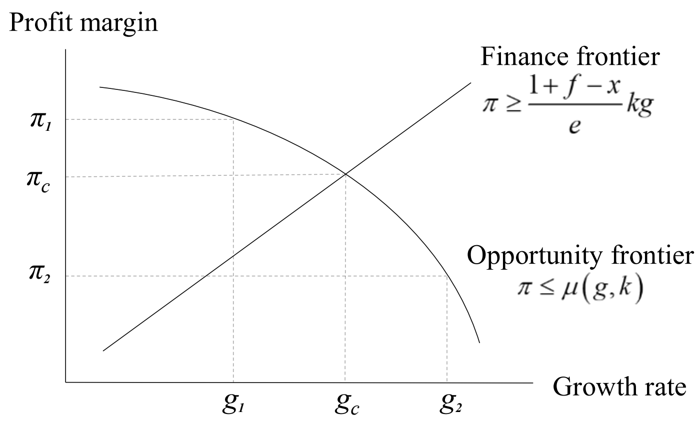

FIGURE 1. CONDITIONS FOR QUANTITATIVE FINANCIAL CONSTRAINT

The finance frontier and opportunity frontier are combined in Figure 1, which is simplified by provisionally taking k to be given. The key choice to be made by a firm pursuing the interests of its shareholders is its position on its opportunity frontier. This frontier is a trade-off between a higher profit margin and faster growth of profits, each of which would benefit shareholders. The chosen position thus depends on the form of this trade-off and on shareholder preferences – especially the rate at which future profits are discounted.

If the shareholder-preferred position on the opportunity frontier is to the left of its intersection with the finance frontier, for example at (π_1, g_1), the firm's investment decisions are financially unconstrained. Its choices of (one or more of) its financial asset ratio, external finance ratio and retention ratio will therefore differ from the limits set by the financial parameters f, x and e in whatever way maximises the benefit to shareholders of its chosen combination of profit margin and growth rate.

If, however, the shareholder-preferred position is to the right of the intersection with the finance frontier, for example at (π_2, g_2), it will be financially unattainable. At the growth rate preferred by shareholders, that is, the maximum attainable profit margin would be too low to finance the required growth of the firm's production capacity. In such a case, the highest attainable growth rate is at the intersection of the frontiers, where shareholders must accept both slower growth, g_c, and a higher profit margin, π_c, than they would ideally like. (In Wood, 1975, which assumes managerial growth maximisation, this intersection defines the only possible outcome.[14])

[14] The conflict between shareholders and growth-maximising managers in a financially constrained firm would be limited to the choice of values for f, x, and e. The main conflict in a financially unconstrained firm, by contrast, would be over position on its opportunity frontier.

The lower-than-desired growth rate, g_c, given k, implies that the firm's level of investment, kg_c, is constrained by the availability of finance to be below the level of investment, kg_2, that would maximise the value of the firm to its shareholders. In other words, a financially constrained firm must have some – maybe many – possible investments that in principle would benefit its shareholders but that in practice it cannot obtain enough money to undertake.[15]

What a financially constrained firm can do for its shareholders, though, is to rank its investment opportunities by their profitability and use its available finance to add as much as possible to the firm's value – as recommended in the capital rationing sections of corporate finance texts (e.g., Brealey et al., 2023, pp. 135-8). The internal rate of return on the least profitable of the investments that the firm can afford to finance is what would be reported as its hurdle rate.

Relaxing the assumption that the investment coefficient, k, is given, the most useful role of the hurdle rate in any firm is often to permit the decentralisation of decisions that affect the value of this coefficient – whether and when to replace older equipment and choosing among more or less capital-intensive designs of larger investments such as new factories. Requiring that a firm's individual investments must exceed some minimum expected rate of return ensures that all of them are consistent with the firm's overall business plan.

The hurdle rate of a financially unconstrained firm should be its external cost of capital. In a constrained firm, however, the hurdle rate measures the internal opportunity cost of alternative investments forgone (which is above its cost of capital). Hurdle rates in the two sorts of firms are thus usually different and vary among financially constrained firms because of differences in their investment opportunities relative to differences in their access to finance.

The scope for variation of hurdle rates is overstated in Figure 1 by assuming the finance frontier to be independent of the opportunity frontier (although both frontiers are normalised, allowing investment opportunities and access to finance to vary in parallel among different-sized firms). Independence is a good approximation for most firms, but not for new firms or unquoted firms in which the supply of new equity responds to investment opportunities. Even in quoted firms, an outward shift of the opportunity frontier can sometimes permit new share issues which also shift the finance frontier outward (by raising x) and keep the hurdle rate down.

2. Determination of hurdle rates and wedges[16]

In this section, the logic of Figure 1 is translated into more specific – but still highly simplified – equations that enable the boundary between financially constrained and unconstrained firms to be precisely defined and the outcomes in both sorts of firms to be more fully described. The hurdle rate and wedge in a constrained firm are derived and their sizes explained. The equations refer to one firm in one long period, but for simplicity without firm and time subscripts.

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[15] Figure 1 differs in many ways from the sort of figure commonly used in other studies to illustrate the effects of financial constraints on investment (e.g., Christodoulou et al., 2022, Figure 1). One important difference is that the present figure treats the available amount of internal finance not as exogenously given but as the endogenous and intended outcome of a constrained firm's growth, profit margin and retention ratio decisions.

[16] This section revises the algebra in Wood (2025) and – crucially – extends it to cover hurdle rates and wedges.

Shareholders are assumed to want to be at the point on the opportunity frontier that maximises the present value of a perpetual net cash flow,

(3) $$V = \frac{R_0(\pi - kg)}{r - g}$$

where R_0 is the initial level of sales revenue, so that $(\pi - kg)$ measures the excess of the firm's annual profits over its investment, which is discounted at the firm's cost of capital, r.[17]

The 'cost of capital' refers as usual to externally supplied capital and r is for simplicity treated as an exogenous constant, unaffected by the amount and composition of the external finance actually used by the firm. Nor is any assumption made about exactly how the cost of capital is defined or measured. A more sophisticated treatment of the cost of capital would not affect the main conclusions of the analysis that follows.

The opportunity frontier is assumed (similarly to Rabinovich, 2021) to be of the form

(4) $$g \leq \alpha - \beta\pi^2,$$

where the parameter α is the firm's growth potential with a zero profit margin, which depends on the nature of its products, markets, and technology. The parameter β defines the extent to which its sales growth would be reduced by a higher profit margin, which is the inverse of its dynamic monopoly power and depends on the availability and relative product quality of other suppliers. To simplify later equations, the effect on the opportunity frontier of the investment coefficient, k, in equation (2) is omitted from equation (4) – but not forgotten.

The shareholder-preferred growth rate, g^*, can be derived straightforwardly as

(5) $$g^* = 2\alpha - r\left[1 + 2k\left[(\alpha - g^*)\beta\right]^{0.5}\right]$$

which unsurprisingly increases with the firm's growth potential and decreases with its cost of capital. The bracketed term that amplifies r is balancing the advantage to shareholders of faster net cash flow growth against the disadvantage of a lower net cash flow level with faster growth because of higher investment spending (which rises with k). A lower β increases the preferred growth rate because it permits faster growth with less reduction of the profit margin.

The associated shareholder-preferred profit margin, π^*, can likewise be derived as

[17] Using net cash flow rather than dividends as the maximand simplifies the analysis because optimal dividend policy is complicated (Moyen, 2004) and contentious (Brealey et al., 2023, p. 983).

(6) $$\pi^* = \left[\frac{r(1+2\beta k\pi^*) - \alpha}{\beta} \right]^{0.5}$$

which is raised by a higher cost of capital and a higher investment coefficient but lowered by greater growth potential (in order to obtain a larger slice of a more rapidly expanding pie). The effect of greater monopoly power (lower β) on the preferred profit margin is ambiguous but paradoxically usually negative.[18]

Whether the shareholder-preferred outcome is attainable depends on the relationship between the parameters that determine a firm's π^* and g^* and its limiting financial parameters (f, x, e). More precisely, attainability depends on whether

(7) $$\frac{\pi^*}{g^*} = \frac{(\tilde{r}-\alpha)^{0.5}}{(2\alpha-\tilde{r})\beta^{0.5}} \geq \frac{1+f-x}{e}k$$

where $\tilde{r} = r\left[1+2k\left[(\alpha-g^*)\beta\right]^{0.5}\right] = r(1+2\beta k\pi^*)$.

The first two terms in (7) are the ratio of the shareholder-preferred profit margin to the preferred growth rate, while the term to the right of the inequality is the minimum financially viable ratio of the firm's profit margin to its growth rate (from the finance frontier in equation (1)). If the preferred ratio is at least equal to this minimum ratio, the shareholder-preferred position on the opportunity frontier is attainable. If the preferred ratio is below this minimum ratio, the firm is financially constrained and the preferred opportunity frontier position is unattainable.

For any given shareholder-preferred outcome, a firm is more likely to be financially constrained if the ratio on the right of the inequality in (7) is larger. A larger minimum required ratio could reflect a higher financial asset ratio, f (due to larger short-term fluctuations in profit), or worse access to external finance (lower x) or internal finance (lower retention ratio, e).

For any minimum required ratio, the likelihood of a financial constraint depends on the middle term of (7), which determines the ratio of π^* (from (6)) to g^* (from (5)).[19] This ratio rises with the cost of capital, which determines the choice of position on a given opportunity frontier, but falls with greater growth potential (higher α), which improves the opportunity frontier. Greater monopoly power (lower β) also improves the opportunity frontier and thus reduces π^*/g^* by lowering $\tilde{r}$, but not by much because β in the denominator pulls the other way.[20]

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[18] The denominator of (6) describes the usual tendency for greater monopoly power to increase the profit margin. This tendency, however, is offset in its numerator by lower β also raising g^* in equation (5) – because it improves the firm's opportunity frontier – and hence reducing net cash flow by requiring higher investment. The extent of the offset depends on the investment coefficient, k, in the amplified r term. With a low enough k, greater monopoly power could still increase π^*, but with normal values of k (well above unity), a lower β reduces π^*.

[19] This expression should be treated with caution. It accurately captures the key qualitative relationships among the variables involved but is a real number only if $2\alpha > r\left[1+2k\left[(\alpha-g^*)\beta\right]^{0.5}\right] > \alpha > g^*$. Within this limited range, moreover, the size of π^*/g^* is unrealistically sensitive to changes in the sizes of the determining variables.

[20] A low k can cause a lower β to raise π^*, but it raises π^*/g^* only with unrealistically low values of k.

The investment coefficient, k, influences both the left and the right of the inequality in (7), and variation in k changes the ratios on both sides in the same direction. A higher k, for example, causes shareholders to prefer a lower growth rate and thus a higher π^*/g^* because net cash flow is reduced by the higher investment needed for a given growth rate. With a given set of limiting financial parameters (f, x, e), a higher k also raises the minimum profit margin needed to finance any given growth rate. The net effect of a higher k on the likelihood of a firm being financially constrained thus depends on the relative size of these two effects.[21]

Less obviously, changes in two other elements on the left and right of the inequality in (7) could also cause the ratios on both sides to move in the same direction. One is the effect mentioned earlier of an unusually favourable opportunity frontier – particularly a high value of α – on the availability of new equity. This effect could raise the external finance ratio, x, and hence reduce the financially required minimum ratio of π to g as well as reducing π^*/g^*.

The other two-sided effect is that changes in the cost of capital could also change the financial parameters. For example, a lower interest rate could raise x by raising target debt/equity ratios or raise e by reducing interest payments out of profits. Lower interest rates might also reduce f (because of a smaller income offset to the opportunity cost of holding reserves) or raise x by increasing share prices and thus making it easier to obtain finance from new issues by reducing the required number of additional shares. A lower cost of capital could thus reduce both the shareholder-preferred and the required minimum ratio of π to g.

As was illustrated in Figure 1, the outcome in a financially constrained firm is governed by the intersection of its opportunity and finance frontiers. Given the form of the opportunity frontier in equation (4), the financially constrained growth rate, g_c, is derived in the Appendix as

(8) $$g_c = \frac{\left(1+4\alpha\beta\varphi^2k^2\right)^{0.5}-1}{2\beta\varphi^2k^2}$$

where φ (so *phi* for 'finance') = $(1 + f - x)/e$ is the minimum required ratio of the firm's profits to its investment. The constrained growth rate increases with a firm's growth potential (α) and with its monopoly power (a lower β).[22] So does the constrained profit margin:

(9) $$\pi_c = \varphi k g_c = \frac{\left(1+4\alpha\beta\varphi^2k^2\right)^{0.5}-1}{2\beta\varphi k}$$

A tighter financial constraint (higher φ), however, has opposite effects on a constrained firm's growth rate and profit margin, lowering g_c but raising π_c.[23]

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[21] The unrealistically sensitive algebra of (7) implies that the derivative of π^*/g^* with respect to k is far higher than the derivative of the minimum financially required ratio of π to g.

[22] The effect of a higher α in equation (8) is obvious. The square root in the numerator of equation (8) causes its numerator to be reduced proportionally less than its denominator by a lower value of β.

[23] Because φ is squared in the denominator of equation (8) but not of equation (9).

Equations (8) and (9) imply that a higher k has the same effect as a higher φ. This is misleading because the simplification earlier of equation (2) into equation (4) neglected the positive effect of a higher k on the opportunity frontier – by lowering production costs (and hence prices) or by enabling the production of new and better outputs. Equations (8) and (9) capture only the negative effect of a higher k through the finance frontier. In reality, a higher k could move the constrained growth rate either way, with g_c being maximised (and closest to the preferred higher g^*) when the net effect on it of a small change in k is zero.[24]

The investment coefficient, k, which greatly affects the firm's level of investment, gk, should thus in principle be modelled as a third endogenous variable (alongside g and π), whose value is determined, like the other two variables, at whatever level maximises the (constrained) value of the firm. That level depends on how k affects the firm's opportunity frontier and on its hurdle rate of return, δ – with lower δ tending to raise k. For an unconstrained firm, the hurdle rate is its cost of capital, r. In a financially constrained firm, the hurdle rate is endogenous.

The hurdle rate in a financially constrained firm can be inferred from the preferred growth rate equation (5). That equation describes the value-maximising relationship among three principal variables: the growth potential parameter α, the growth rate g, and a discount rate r. Equation (5) refers to a financially unconstrained firm, whose growth rate depends on an exogenous cost of capital, but it can be rearranged and relabelled to refer to a constrained firm as

(10) $$\delta = \frac{2\alpha - g_c}{1 + 2k\left[\left(\alpha - g_c\right)\beta\right]^{0.5}},$$

in which the growth rate (from equation 8) is independent of the cost of capital, while the hurdle rate of return, δ, which becomes the relevant discount rate, is endogenous. In other words, δ is the hypothetical discount rate that would cause an unconstrained optimising firm with the same opportunity frontier to choose the same rate of growth (and with an altered k the same level of investment) as in a constrained firm.

A constrained firm's wedge, κ, is thus the excess of this hurdle rate over its cost of capital

(11) $$\kappa = \delta - r = \frac{2\alpha - g_c}{1 + 2k\left[\left(\alpha - g_c\right)\beta\right]^{0.5}} - r\,.$$

Another way of thinking about the wedge is as the difference between the internal opportunity cost and the external opportunity cost of retained earnings – with the former being the return forgone at the margin by not investing them in physical assets and the latter the return forgone by not investing them in financial assets of similar risk – minus the spread added to its cost of capital by financial market imperfections.

Intuitively, the wedge in a constrained firm must relate closely to the Lagrange multiplier of the quantitative financial constraint in the firm's value-maximisation problem. Algebraically,

[24] As is illustrated in a variant of Figure 1 in Wood (1975, p. 89).

with this constraint being $\pi/kg \geq \varphi$, the effect of a small rise in φ on the constrained value of the firm is derived in the Appendix as

$$(12) \qquad \lambda_{\varphi} = R_0 k \left\{ \left(\frac{1}{r/g_c - 1} \right)^2 \left(\frac{r}{\delta} - 1 \right) \right\}$$

The Lagrange multiplier λ_φ is negative because in a constrained firm δ must exceed the cost of capital r, which makes the final bracketed term negative. The (absolute) size of λ_φ increases with the proportional excess of the hurdle rate over the cost of capital, which (since r is given) rises with the wedge ($\delta - r$) and measures the firm's internal scarcity of finance. In a financially unconstrained firm, $\delta = r$, so $\lambda_\varphi = 0$.[25]

G&H (2025 Section VIA), too, show how wedges reduce the value of firms. Their analysis of the investment-reducing effect of hurdle rates above costs of capital is essentially the same as in the present model. However, it treats wedges as exogenous, rather than them being, as in the present theory, the endogenous outcome of quantitative financial constraints.

In summary, the hurdle rate of return is:

$$(13) \qquad \delta = \begin{cases} r \;\; if \;\; \pi^*/g^* \geq \varphi k \\ \dfrac{2\alpha - g_c}{1 + 2k\left[\left(\alpha - g_c\right)\beta\right]^{0.5}} \; if \; \pi^*/g^* < \varphi k \end{cases}$$

and must be greater than the cost of capital, r, when a firm is financially constrained ($\pi^*/g^* < \varphi k$).[26] The hurdle rate and the size of the wedge in a financially constrained firm are determined by the several influences summarised in equations (8), (10) and (11), as follows.

The most important influence is a firm's growth potential, α, which is the main determinant of its need for investment. A higher α would thus raise both δ and κ, though by less than is implied by the initial 2α in equation (10), because it also raises g_c in equation (8).

Greater monopoly power, reflected in a lower value of β, tends to reduce a constrained firm's hurdle rate and wedge by raising both its growth rate and its profit margin. The resulting higher level of profits, given the retention ratio, e, increases a firm's supply of internal finance, which lowers its overall scarcity of finance.

[25] Equation (12) also resolves the apparent paradox that in a period of falling interest rates the rise in wedges as a result of slow adjustment might suggest that finance had become scarcer. What the rise in wedges would actually be showing is that the lower interest rates were increasing the loss of value to shareholders caused by a given set of quantitative financial constraints.

[26] This result, like all the other analysis in this section, assumes firms to be run in the interests of their shareholders. In a firm controlled by growth-maximising managers (as in Wood, 1975), the chosen growth rate would always be g_c, so the chosen δ would be below r if $\pi^*/g^* > \varphi k$.

A looser financial constraint, reflected in a lower value of φ, raises a constrained firm's growth rate and so in equation (11) reduces its hurdle rate and its wedge. The looser financial constraint might reflect a lower f (a smaller required financial cushion), a higher e (larger retainable profit share), or higher x (better access to external finance). In firms or cases where the supply of new equity responded to exceptional investment opportunities, higher x might be caused by higher α, which would reduce the effect of variation in α on both the hurdle rate and the wedge.

As explained earlier, the value of φ could also be affected by variation in the cost of capital – among firms and over time. For example, lower interest rates could raise x or e or reduce f. So the hurdle rates even of financially constrained firms are likely to vary to some degree in the same directions as their costs of capital, making the inverse variation of wedges in response to variation of costs of capital smaller than is suggested by equation (11).

The effect of the investment coefficient, k, on the hurdle rate and wedge is less straightforward than for the other determinants of g_c in equation (8) because k is endogenous. There is also two-way causation, since the hurdle rate is used in decisions that affect the investment coefficient, with a higher δ tending to reduce k. A further complication is that a higher k could increase α by lowering costs or improving products, with a positive effect on δ in equation (10).

These complications make it hard to generalise about the effect on δ and κ of even an exogenous shift in k. They also affect the impact of changes in other determinants of δ. But it seems likely that an exogenous rise in the investment coefficient would raise the hurdle rate and the wedge, contrary to what equations (8), (10) and (11) imply, and that the effects on δ and κ of exogenous changes in α, β, and φ are smaller than is implied by the relevant equations.[27]

The preceding paragraphs refer to the determination of hurdle rates and wedges in financially constrained firms. It is crucial to remember, however, that the determining variables also affect whether or not firms are financially constrained. In other words, the analysis must recognise that exogenous changes in variables that lower the hurdle rate or wedge in a constrained firm will also, if large enough, cause the firm to become unconstrained, as shown by the inequality in (7) above. In such cases, the wedge disappears and the effects of the determining variables on growth, investment and profit margins often become different.

3. Consistency with Gormsen and Huber results

How well does the theory in the previous two sections fit the evidence presented by Gormsen and Huber (2025)? There is evidently broad consistency, since the theory can explain why – as in the G&H data – wedges are common and vary among firms. Also important, however, is to consider whether there is consistency in detail and whether the present theory can explain other G&H results better than standard theory.

[27] *Reasoning*: Assuming $\alpha = \alpha(k)$ with $\alpha' > 0$, an exogenous rise in k would initially increase δ, because the positive effect via the 2α term in (10) would exceed any possible negative effect of a rise in the g_c term (on which k and α have conflicting effects). This initial positive effect would be reduced but not reversed by the subsequent fall in k as a result of the rise in δ. If so, the feedback from the effects on δ of changes in the other parameters in equation (8) would be negative: e.g., an initial rise in δ would lower k, which would tend to lower δ and thus to make the eventual size of the change in δ smaller than equations (8) and (10) predict.

One example of better explanation concerns the length of the lags in the G&H estimates of the adjustment of hurdle rates to changes in costs of capital. It is challenging in standard theory to explain lags of more than a decade, given the rapidity of responses to opportunities for gain in other areas of financial activity. The present theory, however, predicts long lags.

In financially unconstrained firms, whose costs of capital directly affect their positions on their opportunity frontiers (equations (5) and (6)), the present theory predicts that adjustment should occur as soon as firms become confident that changes in the cost of capital are enduring rather than transitory. The position of a constrained firm on its opportunity frontier, however, depends on the location of its finance frontier. Changes in the cost of capital tend indirectly to change the hurdle rates of constrained firms in the same directions by shifting their finance frontiers, but the adjustment is certain to be slow.

This is because a change in interest rates has little immediate effect on the amount of interest being paid by constrained firms, which depends mainly on the rates on their outstanding long-term debt and whose short-term debt is offset by short-term financial asset cushions. There will thus be little pressure through either of the main channels through which a rise (say) in interest rates could make a firm's financial frontier more restrictive, namely (i) reducing the external finance ratio (x) by increasing the risks of borrowing and (ii) reducing the retention ratio (e) by increasing the amount of interest paid out of profits. As a firm's long-term debt matures and is refinanced, the effects of a higher interest rate on its finance frontier become larger, but it may be ten years or more before the refinancing is complete.

Nor does the present theory predict that the change in the hurdle rate after full adjustment will generally be the same size as the change in the interest rate. A firm's cost of capital is one of several determinants of the restrictiveness of its finance frontier, which in turn is one of several determinants of its constrained growth rate and thus of the size of the wedge between its hurdle rate and its cost of capital. There is thus likely to be wide cross-firm dispersion of the eventual size of changes in hurdle rates in response to changes in the cost of capital – consistent with what G&H estimate and report in their Figure 4.

Moreover, although a change in the cost of capital alters the size of a firm's wedge during the period of adjustment, the present theory provides no reason to suppose that the size of its wedge at the start or the end of the adjustment process will be zero. G&H's position on this point is not entirely clear. In G&H (2025) they propose some non-temporary reasons for wedges but also imply (for example in the abstract) that most wedges are temporary. Moreover, their long-run analysis of investment in G&H (2025a, section 4.1) is all about the influence of costs of capital and does not even mention the word 'wedge'.

Wedges are assumed away in G&H (2025a) on the grounds that (i) hurdle rates eventually react to changes in costs of capital, as shown in G&H (2025), and (ii) investment rates are inversely correlated across firms with their costs of capital (2025a Table 2). Neither piece of evidence, however, is inconsistent with wedges being an enduring and important feature of reality. The present theory also predicts that hurdle rates eventually react to changes in costs of capital even in financially constrained firms and that investment rates vary among constrained firms with their costs of capital (since hurdle rates must be at or above costs of capital).

Of the three non-temporary reasons for wedges in G&H (2025), two – and the evidence that supports them – are consistent with the present theory. One of these is firm-specific risks that are known to managers but are not known to financial markets. In the present theory higher risks of this sort will induce managers to choose higher financial asset ratios (f) and/or lower external finance ratios (x), increasing the likelihood of their firms being financially constrained and if so of having larger wedges, as is the case in G&H Table A10.

The second reason, which G&H label 'financial constraints', is also consistent with the present theory but refers to a different and much rarer sort of constraint than in the present theory. G&H associate financial constraints with short-term cash-flow problems caused by bad luck or bad management. Such problems are most frequent in firms that are small or young, so the G&H financial constraint variable is an index relating the probability of cash-flow problems to firm size and age (from Hadlock and Pierce, 2010), which is significantly related across firms to the sizes of their wedges (G&H, Table A10).

The present theory, by contrast, sees financial constraints not as a rare and acute problem but as a normal feature of the operation of many successful and financially secure firms. Though small and young firms are usually more tightly constrained than big and well-established firms (e.g., Christodoulou et al., 2022, Figure 6), firms of all sizes and ages face upper limits on the amounts of finance for investment that they can access within their planning horizons. Firms for which these limits are not binding constraints select investments on the basis of their costs of capital. In firms where the limits bind, investment decisions are optimised subject to these quantitative constraints by setting hurdle rates above costs of capital.

The present theory could thus help to solve the puzzle of why, although most corporations use hurdle rates, only 3% of them tell investors what these rates are (G&H pp. 2018-9). All chief financial officers (CFOs) and most professional investors have read or been told or taught that the financial welfare of shareholders requires hurdle rates to equal costs of capital. This widely shared belief puts the CFOs of financially constrained firms in a difficult position: their wedges are optimal for shareholders and setting their hurdle rates equal to costs of capital would lead to disaster, but for them to explain that this was because of limited access to finance would ring alarm bells about their firms' solvency. Best, therefore, to keep quiet about the wedges and, if forced to reveal them, then to invent some other plausible-sounding explanation, as many CFOs in G&H seem to do – with only one of those quoted (p. 2041) daring to tell the truth.

The third G&H non-temporary reason for wedges conflicts with the present theory, which also offers an interpretation of G&H's supporting evidence that is more consistent with other recent research on firms. This reason is that wedges reflect wrong investment decisions in firms that can survive their mistakes only because they have monopoly market power, with the supporting evidence being a positive correlation between wedges and profit margins (Table A10).

By contrast, in equations (8)-(11) of the section 2 model the relationship between wedges and monopoly power (measured as the reciprocal of the parameter β) is negative. Greater monopoly power improves a financially constrained firm's opportunity frontier, which tends to raise its growth rate. Because this higher growth rate is closer to the financially unconstrained growth rate, the financial constraint binds less tightly and so the wedge – which reflects the tightness of the constraint – is smaller.

The model in section 2 is consistent with the G&H finding of a positive cross-firm correlation between wedges and profit margins. This association, however, is caused by variation not in β but in the growth potential parameter, α, of which a higher value increases both a constrained firm's profit margin (equation (9)) and its wedge (through the 2α term in equation (11)).

The present theory can also explain why the regression of wedges on profit margins (and other variables) in G&H Table A10 explains so little of their cross-firm variation, which is the result of its estimation across all firms, not just financially constrained ones. In the section 2 model, the wedges of unconstrained firms are zero and their profit margins usually vary inversely with their monopoly power (equation (6)), rather than directly as in constrained firms (equation (9)). The mixture of constrained and unconstrained firms also muddies the influence of variation in α on the statistical relationship between wedges and profit margins because, as equation (6) also shows, in a financially unconstrained firm a higher (say) α tends to lower the profit margin, rather than, as in a constrained firm, to increase it (equation (9)).

The G&H interpretation and this paper's interpretation of the positive statistical association between wedges and profit margins rest on different assumptions about the main cause of cross-firm variation in profit margins. G&H assume this to be variation in monopoly power, while the present theory assumes it to be variation in growth potential generated by differences in productivity and managerial quality.

Other research seems more supportive of the present interpretation. Studies of the rise in the average profit margin show that changes in technology and anti-trust policy have increased the monopoly power (and the size) of more profitable firms within industries, but that the starting point for this process was their higher initial levels of productivity (e.g., Autor *et al*., 2020; De Loecker *et al*., 2020). The huge mass of theoretical work on heterogeneous firms is similarly motivated by strong evidence of wide cross-firm variation in productivity. Across industries, moreover, there is a research history of weak or small inverse relationships between average profit margins and the intensity of competition as measured by market structure or the severity of competition policy (Melmiès, 2015).

Vital to the main argument of G&H is their evidence (2025, section V) of an inverse empirical relationship between firms' hurdle rates and their levels of investment, consistent with both standard theory and the present theory. G&H find an inverse relationship also between wedges and investment, which they see as a malign effect of monopoly power (p. 2044, Table A15) and is harder to compare with the present theory because its estimation treats wedges as exogenous, whereas in the present theory they are endogenous.

The model in section 2 could generate an inverse association between wedges and investment, but its economic interpretation would be different. Other things being equal, a less favourable finance frontier (higher φ) would raise a constrained firm's wedge and lower both its growth rate and its investment coefficient. Greater growth potential (higher α) would raise both its wedge and its growth rate, although the positive effect on its investment of faster growth would be to some degree offset by a lower investment coefficient.

The present theory also offers an unusual answer to the question of whether and when managers should care about wedges. In financially unconstrained firms, wedges should not exist and are inflicting costs on shareholders. In financially constrained firms, though, wedges are irrelevant – all that should matter to shareholders is setting the right (constrained) hurdle rate. How far it is above the firm's cost of capital should not concern managers – which may help explain their often inaccurate estimates of the cost of capital (G&H 2025a) – except for the diplomatic issue of having to explain to shareholders why they are not following the textbook rules.

4. Conclusions

The empirical research of Gormsen and Huber (2025) is a major addition to knowledge about the hurdle rates that firms use in investment decisions. The inconsistency of their results with the standard theory of corporate finance also provides an incentive to improve the theory.

This paper argues that the inconsistency reflects successful and inconspicuous management by many firms of persistent quantity constraints on access to finance. On that basis it has set out a non-standard theory that could explain the G&H results. Its policy implications conflict with those of the G&H 'managerial mistakes' interpretation of their results.

This theory was formalised in a simple diagram (section 1) and an algebraic model (section 2). The model is also highly simplified and could be developed further in many ways, for example with alternative forms of the valuation function, opportunity frontier and finance frontier, and with extra equations to cover links discussed here only verbally. More radically, 'investment' could be redefined to include intangible assets, though this would require redefinition also of the profit margin and other departures from standard accounting conventions.

The consistency of the theory with empirical evidence was assessed above by using the results presented in G&H (2025). An obvious next step would be take the theory to the G&H data and make predictions that could be compared with their results – for instance, the strategic financial constraints explanation of high hurdle rates versus their monopoly power explanation.[28] The legal inaccessibility of their data, however, ruled out this next step for the present paper.

In the future, the G&H data may become available, or they may use it to test the present theory. For now, the best option for other researchers, though it is beyond the scope of this paper, may be to compare the theory's predictions systematically with other evidence, especially the many

[28] Key elements of this work would be identification of the opportunity and finance frontiers and distinguishing between financially constrained and unconstrained firms. The complication of two categories of firms might be addressed with the Heckman correction, since the effects of changes in exogenous variables on the probability of a firm being financially constrained are in the same directions as on the size of a constrained firm's wedge.

studies of relationships between financial constraints, cash flow and investment.[29] Their results might be reinterpreted in light of the present theory's distinction between long-run and short-run financial behaviour, its specification of the difference between financially constrained and unconstrained firms, and its treatment of long-term profit margins as endogenous.

[29] Hubbard (1998), Moyen (2004), and Christodoulou et al. (2022) provide extensive references and discussion. A recurring question has been whether the sensitivity of investment to cash flow is a good measure of the degree to which a firm is financially constrained, a proposition contested by Kaplan and Zingales (1997).

APPENDIX

Derivation of constrained growth rate and Lagrange multiplier

Maximise the discounted present value of a perpetual net cash flow (from equation (3))

$$V = \frac{R_0\left(\pi - kg\right)}{r-g}$$

for given R_0 and r, subject to the finance and opportunity frontiers (from equations (1) and (4)),

$$\frac{\pi}{kg} \geq \varphi \text{ where } \varphi = \frac{1+f-x}{e}$$

$$g \leq \alpha - \beta\pi^2 \text{ and thus } \pi\left(g\right) \leq \left(\frac{\alpha - g}{\beta}\right)^{0.5}$$

with the Lagrangian function being

$$L = \frac{R_0\left(\pi\left(g\right) - kg\right)}{r-g} + \lambda\left(\frac{\pi\left(g\right)}{kg} - \varphi\right).$$

To obtain the maximum constrained growth rate, derive the first order condition

$$\frac{\partial L}{\partial \lambda} = \left(\frac{\pi\left(g\right)}{kg} - \varphi\right) = 0$$

implying

$$g = \frac{1}{\varphi k}\left(\frac{\alpha - g}{\beta}\right)^{0.5}$$

whose explicit solution is

$$g_c = \pm\frac{\left(1+4\alpha\beta\varphi^2k^2\right)^{0.5} - 1}{2\beta\varphi^2k^2}$$

of which only the positive value is economically relevant, as in equation (8) – and can also be derived by solving equations (1) and (4) simultaneously.

To obtain the Lagrange multiplier, derive the first order condition

$$\frac{\partial L}{\partial g}=\frac{\partial}{\partial g}\left(\frac{R_0\left(\pi(g)-kg\right)}{r-g}\right)+\frac{\partial}{\partial g}\lambda\left(\frac{\pi(g)}{kg}-\varphi\right)=0$$

where

$$\frac{\partial}{\partial g}\left(\frac{R_0\left(\pi(g)-kg\right)}{r-g}\right)=R_0\left\{\frac{-0.5\left[\beta(\alpha-g)\right]^{-0.5}}{r-g}+\frac{\left[(\alpha-g)\beta^{-1}\right]^{0.5}}{(r-g)^2}-\frac{rk}{(r-g)^2}\right\}$$

$$\frac{\partial}{\partial g}\lambda\left(\frac{\pi(g)}{kg}-\varphi\right)=-\frac{\lambda}{k}\left\{\frac{(\alpha-g)^{0.5}\left[1+0.5g(\alpha-g)^{-1}\right]}{\beta^{0.5}g^2}\right\}$$

so that manipulation of the first order condition yields

$$\lambda=R_0k\left\{\frac{\left(0.5\left[\beta(\alpha-g)\right]^{-0.5}[r-g]-\left[(\alpha-g)\beta^{-1}\right]^{0.5}+rk\right)\beta^{0.5}g^2}{[r-g]^2[\alpha-g]^{0.5}\left(1+0.5g[\alpha-g]^{-1}\right)}\right\}$$

which simplifies to

$$\lambda=R_0k\left\{\left(\frac{1}{r/g-1}\right)^2\left(\frac{r\left(1+2k\left((\alpha-g)\beta\right)^{0.5}\right)}{2\alpha-g}-1\right)\right\}$$

which because, from equation (10)

$$\delta=\frac{2\alpha-g_c}{1+2k\left[(\alpha-g_c)\beta\right]^{0.5}},$$

can be rewritten (as in equation (12)) as

$$\lambda_\varphi=R_0k\left\{\left(\frac{1}{r/g_c-1}\right)^2\left(\frac{r}{\delta}-1\right)\right\}.$$

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